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# Employing the Rayleigh Distribution to Estimate the Reliability of a French Fries Cutting Machine and Using a Markov Chain to Estimate the Probability of Transitioning from One State to Another for the Short and Long Term

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**Abstract:** In this study we used two methods to estimate the hazard of the cutting machine of French fries which are reliability analysis with Rayleigh distribution and Markov chine to estimate the probability transition from a state to another, the data that had been used were 70 observations of failure time which has been taken from Bazian manufacturer for producing French fries, after computing the reliability analysis we transform the real data above into the dummy variable based on the arithmetic mean which is calculated from the real failing time data (Mean = 46.53 min) as when the failing time below 46.53min takes zero otherwise takes one, and the transition frequency matrix has been calculated, the results presents that the Rayleigh distributions probability density function exhibits a decreasing trend over time, a substantial portion of failure times the system failure is most likely to occur between 10 and 80 minutes, marking a critical period. The probability density function significantly decreases after 120 minutes. The reliability curve shows a decline over time; the likelihood of the system remaining reliable beyond 40 minutes is about 0.6, while the probability drops to 0.2 for reliability beyond 80 minutes. Additionally, there is a 51.86% chance that a failure will occur within 46.53 minutes and be followed by another failure within the same timeframe. Similarly, there is a 51.86% chance that a failure occurring after 46.53 minutes will be followed by another failure occurring after the same duration value is 0.4814.

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# توظيف توزيع رايلي لتقدير المعولية لآلة تقطيع البطاطس المقلية واستخدام سلسة ماركوف لتقدير احتمالية الانتقال من حالة إلى أخرى على المدى القصير والطويل الأجل


| اسراء سعدون علوان | هندرين عبدالله طاهر | هيشو عثمان |
|---|---|---|
| كلية ادارة والاقتصاد | كلية التجارة | كلية ادارة والاقتصاد |
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**المستخلص**

في هذه الدراسة، استخدمنا اثنين من الأساليب لتقدير خطر آلة قطع البطاطس المقلية، وهي تحليل الموثوقية باستخدام توزيع رايلي وسلسلة ماركوف لتقدير احتمالية الانتقال من حالة إلى أخرى. تم استخدام بيانات تتضمن 70 مشاهدة لأوقات الفشل، والتي تم الحصول عليها من مصنع بازيان لإنتاج البطاطس المقلية. بعد حساب تحليل الموثوقية، قمنا بتحويل البيانات الحقيقية المذكورة أعلاه إلى متغيرات وهمية استنادًا إلى المتوسط الحسابي الذي يتم حسابه من بيانات الوقت الحقيقي للفشل (المتوسط = 46.53 دقيقة)، حيث يتم أخذ قيمة صفر عندما يكون وقت الفشل أقل من 46.53 دقيقة وإلا يتم أخذ القيمة واحد، وتم حساب مصفوفة الترددات للانتقال. تبين النتائج أن دالة كثافة الاحتمال لتوزيع رايلي تظهر انخفاضًا تدريجيًا مع مرور الوقت، حيث تتركز نسبة كبيرة من أوقات الفشل ضمن النطاق من 10 إلى 80 دقيقة، مما يسلط الضوء على نافذة زمنية حرجة لفشل النظام المحتمل. تنخفض دالة كثافة الاحتمال بشكل أكبر بعد مرور 120 دقيقة من الزمن. وتتراجع منحنى الموثوقية مع مرور الوقت، حيث تبلغ فرصة الموثوقية لأكثر من 40 دقيقة تقريبًا 0.6، ومع ذلك، فإن فرصة الموثوقية تنخفض أكثر عند تجاوز 80 دقيقة تقريبًا وتصبح 0.2. من الجدير بالذكر أن احتمال حدوث فشل أقل من 46.53 دقيقة والفشل الذي يليه أقل من القيمة المذكورة أعلاه هو 0.5186، واحتمال حدوث فشل أكثر من 46.53 دقيقة والفشل الذي يليه أكثر من القيمة المذكورة أعلاه هو 0.4814


**الكلمات المفتاحية:** نظرية الاحتمالات، تحليل المصداقية، توزيع رايلي وسلسلة ماركوف.

## 1-1. Introduction

French fries, a beloved side dish or snack, are made from potatoes cut into elongated strips and deep-fried until crispy. They're known for their golden-brown color, crispy exterior, and fluffy interior. The origin of French fries is debated, with claims of their creation from both Belgium and France. The process involves slicing potatoes into sticks or strips, which are then fried in oil until they achieve a crispy texture. The outer layer crisps up due to the oil, while the inside remains soft and starchy. French fries are commonly seasoned with salt, and they're often served with various accompaniments like ketchup, mayonnaise, cheese, gravy (as in poutine), or other sauces. They are a staple in fast-food chains worldwide and are also frequently homemade. The popularity of French fries is immense, with diverse variations in size, shape, and seasoning across different cultures.

They are enjoyed as a side dish, snack, or complement to main courses, remaining a universally loved comfort food. We examined the Bazian manufacturer, which produces French fries using a production line composed of various interconnected parts. Our focus was on the cutter machine, which processes three tons of potatoes per hour after they are prepared by preceding stages. Over 21 months of operation, this machine experienced 70 failures, impacting output. We analyzed maintenance data to assess the machine's reliability and identify the causes of these failures.

**1-2. Literature review:** Ibrahim & Ismail (2022) They analyzed monthly time series data from January 2005 to September 2019 to assess the financial and monetary impact of inflation on the Iraqi economy. It focused on key variables such as public spending, broad money supply (M2), and public debt to understand the decisions made by policymakers during economic shocks and crises. Due to Iraq's reliance on crude oil for public spending, fluctuations in international oil prices posed challenges and led to structural failures in the time series, complicating the measurement of inflation's effects. The study employed Bai-Perron tests to identify structural failures and used Markov models to analyze the financial and monetary effects during different economic conditions. Significant structural imbalances were observed, prompting the study to identify and classify structural changes as temporary or permanent shocks. The study recommended improved coordination between monetary and fiscal policymakers to achieve better economic outcomes, emphasizing the importance of stable and harmonized policies in reducing inflation rates and fostering overall economic stability.

Hamdin & Hussein (2023) they investigate earthquake occurrence probabilities and estimates earthquake risk states in Iraq, located at the northeastern corner of the Arabian Plate, which collides with the Iranian (Eurasian) Plate, forming the active Zagros seismic belt. Utilizing a wavelet Markov chain model, the study analyzes the transition probability between earthquake occurrence states. Data from the Earth Scope website spanning January 2013 to November 2022 were examined. Results indicate that after 115 months, the likelihood of an earthquake occurrence not being felt or being felt rarely is 0.009, while the probabilities of it being felt slightly, frequently, and by the majority of people in the affected area were 0.620, 0.124, and 0.237 respectively. Additionally, the perceived probability of a damaging earthquake occurring, affecting poorly constructed buildings, was 0.008.

Batah & Kibria (2024) they explored parameter estimation and reliability systems within the stress-strength model, assuming independence between variables X and Y, both following a three-parameter Weibull-exponential Rayleigh distribution (WERD). It derives single, parallel, and series systems for WED. Various estimation methods including maximum likelihood estimator (MLE), exact method of moment's estimator (EMME), weighted least squares estimator (LSE), and shrinkage function (Shf) estimation methods are analyzed for their parameters.

Sadia et al. (2023) they investigated the estimation of a stress-strength reliability model (R = P(X > Y)) utilizing the inverted exponentiated Rayleigh distribution under a unified progressive hybrid censoring scheme (unified PHCS). Maximum likelihood estimates of the unknown parameters are obtained through the stochastic expectation-maximization algorithm (stochastic EMA), along with the creation of asymptotic confidence intervals. Additionally. The paper presents the use of the Gibbs sampler combined with the Metropolis-Hastings algorithm to calculate Bayes estimates and credible intervals under squared error, Linex, and generalized entropy loss functions. Comprehensive simulations are performed to evaluate the efficacy of the proposed estimation methods. Additionally, the paper emphasizes the significance of applying reliability studies in diverse fields, such as engineering, by using droplet splashing data under varying nozzle pressures as an example to demonstrate the theoretical results.

Based on the historical overview in this study, some researchers have used reliability analysis while others have employed Markov models. Therefore, we have combined both approaches: the first to estimate reliability and the second to determine the probability of transitioning from one state to another over short and long terms.

**2-1. Rayleigh Distribution:** The Rayleigh distribution, named after Lord Rayleigh, is a continuous probability distribution that is widely used in engineering, physics, and various scientific disciplines. It describes the distribution of magnitude, typically positive and representing the magnitude of a two-dimensional vector whose components are independent and identically distributed. One of the most common applications of the Rayleigh distribution is in modeling the distribution of wind speeds, wave heights.

And other phenomena in nature where the resultant magnitude is influenced by multiple random factors (Ferreir & SiLvA 2017: 312) mathematically, the Rayleigh distribution is characterized by a probability density function (PDF) that reaches its peak at zero and then gradually decreases, forming a right-skewed curve. It is a useful tool for analyzing the variability of magnitudes in various scenarios, especially where there's a cumulative effect of multiple random components. In essence, the Rayleigh distribution provides insights into the probability of different magnitudes occurring within a system or phenomenon, making it a valuable tool in modeling and understanding various natural and engineered processes.

$$f(x) = \frac{x-\gamma}{\delta^2} \, exp \left[ -\frac{1}{2} \left(\frac{x-\gamma}{\delta}\right)^2 \right] \quad (1)$$

$$F(x) = 1- exp \left[ -\frac{1}{2} \left(\frac{x-\gamma}{\delta}\right)^2 \right] \quad (2)$$

$$R(x) = exp \left[ -\frac{1}{2} \left(\frac{x-\gamma}{\delta}\right)^2 \right] \quad (3)$$

$$h(x) = \frac{x-\gamma}{\delta^2} \quad (4)$$

The above equation from 1 to 4 are the probability mass function, distribution function F(x), Reliability function R(x) and hazard function h(x) respectively. Where $\gamma$ is the location parameter and $\delta^2$ is the scale parameter. (Srinivasa et al (2013:258).

**2-2. Markov chains:** The Markov chains is rooted in the concept of stochastic processes and probability theory. A Markov chain is a type of stochastic process—a sequence of random variables representing a system that undergoes transitions from one state to another, following a specific set of probabilistic rules. (Marwf et al (2021) p331),(Taha, Mohammad (2023:251)

Let $\{X_n, n \geq 0\}$be a homogeneous Markov chain with a discrete state space S = {0,1,2,...}. Then the one-step transition probability from state *i* to state *j* is defined by:

$$P_{ij} = P\{X_{n+1} = j / X_n = i\} \qquad i \geq 0, j \geq 0$$

Which is the same for all values of n (as the Markov chain is homogeneous). The transition probability matrix (TPM) of the process $\{X_n, n \geq 0\}$ is defined by

$$P = [p_{ij}] \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{12} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}$$

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Where the transition probabilities (elements of $P$) satisfy

$$p_{ij} \geq 0, \sum_{j=0}^{\infty} p_{ij} = 1 \; for \; i,j = 0,1,2,\ldots$$

If the state space $S$ is finite and is equal to {1,2,...,m}, then $P$ is a square matrix of order $m$, i.e

$$P = [p_{ij}] \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{12} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}$$

Where the transition probabilities (elements of $P$) satisfy

$$p_{ij} \geq 0, \sum_{j=1}^{\infty} p_{ij} = 1 \; for \; i,j = 1,2,\ldots,m$$

We note that a square matrix $P$ whose elements satisfy Eq. (5.32) or (5.33) is called a Markov matrix or stochastic matrix (Wang, B. X., Yu, K., & Sheng, Z. (2014) p 810) (Taha, Mohammad 2023:3130).

**2-3. Stationary Distribution for a Markov Chain:** Let $\{X_n, n \geq 0\}$ i be a homogeneous Markov chain with transition probability matrix $P$. If there exists a probability vector $\pi$ such that

$$\pi P = \pi \qquad (5)$$

Then $\pi$ is called a stationary distribution or steady-state distribution for the Markov chain.

**2-3-1. Components of Markov Chains:** States: A set of distinct conditions or situations the system can be in. These states represent the possible configurations of the system at any given time.

Transition Probabilities: These probabilities determine the likelihood of moving from one state to another in a single step. They are often organized into a transition matrix or diagram. In a Markov chain, the transition probabilities describe the probability of moving from one state to another in a single step. Mathematically, these probabilities are usually represented as elements in a transition matrix P, where Pij is the probability of transitioning from state i to state j. (Starling et al., 2021: 86).

Markov Property: The future state of the system depends only on the current state, not on the sequence of events that led to the current state. This property is known as memory looseness.

**3. Applications:** We used a sample of 70 observations of the failure time To estimate the reliability of the cutting machine, firstly we should detect the real distribution of the data under consideration, for this purpose the goodness of fit has been used, secondly we assumed that there are two state below and over the arithmetic mean of the data which was 46.53 minutes to calculate the transition probability by using Markov chain, the results are shown below:

This application was implemented using the ready-made programs MATLAB and EasyFit.

Table (1): Goodness of Fit Test for Rayleigh Distribution(By authors)

| Test | statistic | P-value | Sample size |
|---|---|---|---|
| Kolmogorov-Smirnov | 0.10457 | 0.40079 | 70 |
| Anderson-Darling | 0.91066 | 0.40012 | 70 |

Summing to up table, the data under study has Rayleigh 2P Distribution the p-value is greater than 0.05 for both tests Kolmogorov-Smirnov and Anderson-Darling.

Table (2): Rayleigh 2P Distribution (By authors)

| Parameters | Scale | Location |
|---|---|---|
| Estimated value | 47.631 | 8.774 |

The above table represents the estimated parameters of the Rayleigh distribution, the scale and the location parameters are equal to (47.631 and 8.774) respectively.

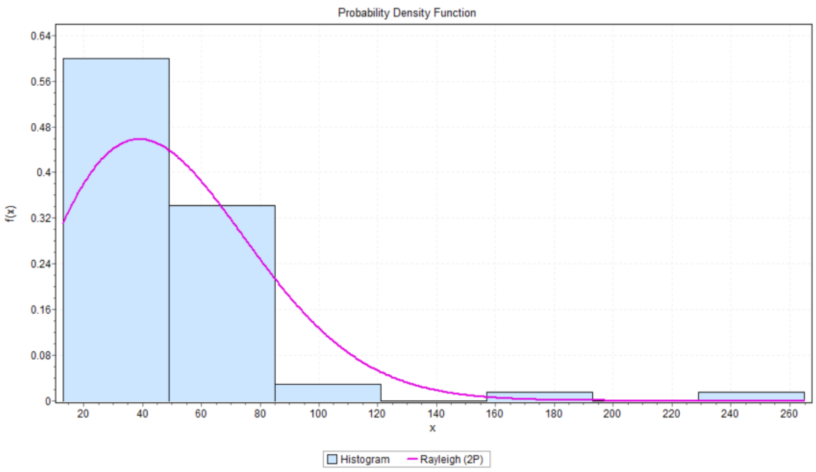


Figure (1): shows the probability density function curve (Easyfit 5.5)

The Rayleigh distributions probability density function exhibits a decreasing trend over time, a substantial portion of failure times concentrates within the range of 10 to 80 minutes, highlighting a critical time window for potential system failure. Probability density function most decreased after 120 minutes period time)Ahmed et al (2020:5).

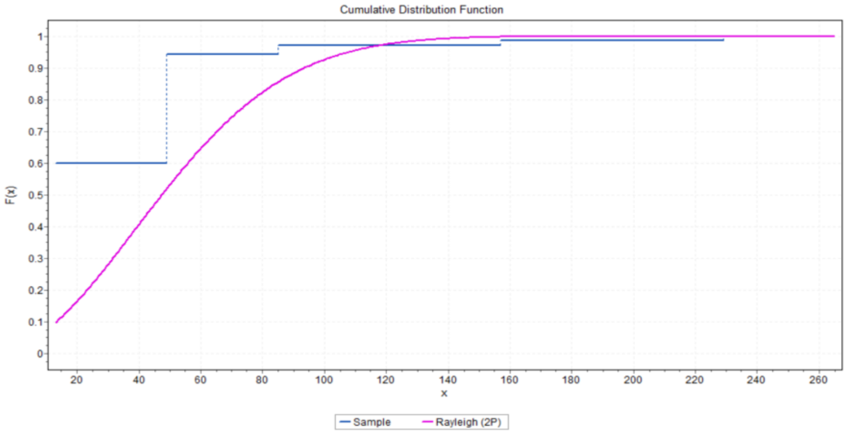


Figure (2): represents the distribution function curve (Easyfit 5.5)

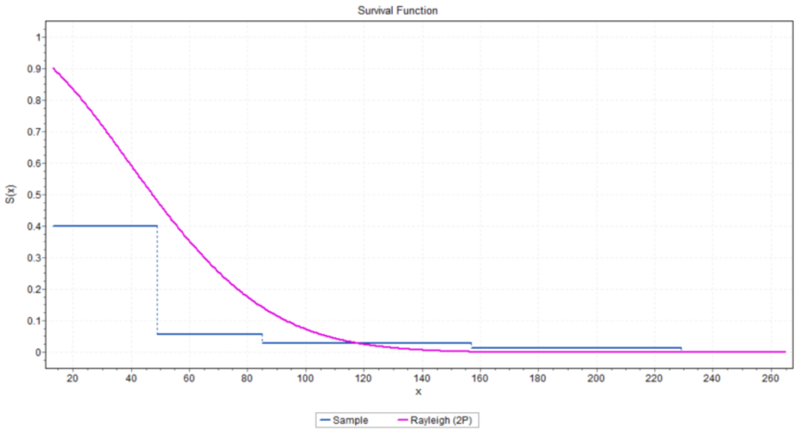


Figure (3): demonstrates the reliability curve (Easyfit 5.5)

The figure (3) shows the reliability curve which is decreasing over time, the chance of the reliability over 40 minutes approximately is 0.6, however, the chance of reliability more decreased over 80 minutes approximately is 0.2.

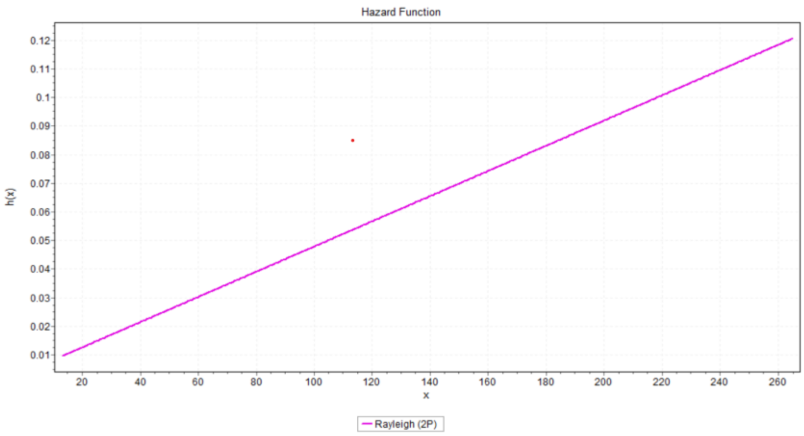


Figure (4): clarifies the hazard function curve (Easyfit 5.5)

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The hazard function curve increasing linearly this implies the hazard on this machine is reach to the peak monotonically and linearly.

Table (3) presents the computed values of f(x), F(x), R(x) and h(x) (By authors)

| No. | f(x) | F(x) | R(X) | h(x) | No. | f(x) | F(x) | R(X) | h(x) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0064 | 0.0498 | 0.9502 | 0.0067 | 36 | 0.0097 | 0.7031 | 0.2969 | 0.0327 |
| 2 | 0.0067 | 0.0564 | 0.9436 | 0.0072 | 37 | 0.0115 | 0.5856 | 0.4144 | 0.0279 |
| 3 | 0.0060 | 0.0436 | 0.9564 | 0.0063 | 38 | 0.0127 | 0.3756 | 0.6244 | 0.0204 |
| 4 | 0.0031 | 0.0114 | 0.9886 | 0.0032 | 39 | 0.0123 | 0.5018 | 0.4982 | 0.0248 |
| 5 | 0.0019 | 0.0039 | 0.9961 | 0.0019 | 40 | 0.0127 | 0.4265 | 0.5735 | 0.0221 |
| 6 | 0.0127 | 0.3629 | 0.6371 | 0.0199 | 41 | 0.0113 | 0.6084 | 0.3916 | 0.0288 |
| 7 | 0.0072 | 0.8130 | 0.1870 | 0.0384 | 42 | 0.0126 | 0.3375 | 0.6625 | 0.0191 |
| 8 | 0.0117 | 0.5740 | 0.4260 | 0.0274 | 43 | 0.0111 | 0.1934 | 0.8066 | 0.0138 |
| 9 | 0.0044 | 0.0228 | 0.9772 | 0.0045 | 44 | 0.0123 | 0.2876 | 0.7124 | 0.0173 |
| 10 | 0.0027 | 0.0085 | 0.9915 | 0.0027 | 45 | 0.0109 | 0.1824 | 0.8176 | 0.0133 |
| 11 | 0.0019 | 0.0039 | 0.9961 | 0.0019 | 46 | 0.0099 | 0.1407 | 0.8593 | 0.0116 |
| 12 | 0.0023 | 0.0060 | 0.9940 | 0.0023 | 47 | 0.0121 | 0.2632 | 0.7368 | 0.0164 |
| 13 | 0.0019 | 0.0039 | 0.9961 | 0.0019 | 48 | 0.0122 | 0.2753 | 0.7247 | 0.0168 |
| 14 | 0.0060 | 0.0436 | 0.9564 | 0.0063 | 49 | 0.0117 | 0.2275 | 0.7725 | 0.0151 |
| 15 | 0.0031 | 0.0114 | 0.9886 | 0.0032 | 50 | 0.0126 | 0.3375 | 0.6625 | 0.0191 |
| 16 | 0.0036 | 0.0148 | 0.9852 | 0.0036 | 51 | 0.0119 | 0.5504 | 0.4496 | 0.0265 |
| 17 | 0.0044 | 0.0228 | 0.9772 | 0.0045 | 52 | 0.0122 | 0.5263 | 0.4737 | 0.0257 |
| 18 | 0.0064 | 0.0498 | 0.9502 | 0.0067 | 53 | 0.0127 | 0.4138 | 0.5862 | 0.0217 |
| 19 | 0.0117 | 0.2275 | 0.7725 | 0.0151 | 54 | 0.0127 | 0.3883 | 0.6117 | 0.0208 |
| 20 | 0.0088 | 0.1032 | 0.8968 | 0.0098 | 55 | 0.0126 | 0.4644 | 0.5356 | 0.0235 |
| 21 | 0.0081 | 0.0862 | 0.9138 | 0.0089 | 56 | 0.0081 | 0.0862 | 0.9138 | 0.0089 |
| 22 | 0.0064 | 0.0498 | 0.9502 | 0.0067 | 57 | 0.0102 | 0.1507 | 0.8493 | 0.0120 |
| 23 | 0.0048 | 0.0274 | 0.9726 | 0.0049 | 58 | 0.0127 | 0.4138 | 0.5862 | 0.0217 |
| 24 | 0.0056 | 0.0378 | 0.9622 | 0.0058 | 59 | 0.0122 | 0.2753 | 0.7247 | 0.0168 |
| 25 | 0.0031 | 0.0114 | 0.9886 | 0.0032 | 60 | 0.0121 | 0.2632 | 0.7368 | 0.0164 |
| 26 | 0.0044 | 0.0228 | 0.9772 | 0.0045 | 61 | 0.0126 | 0.3249 | 0.6751 | 0.0186 |
| 27 | 0.0019 | 0.0039 | 0.9961 | 0.0019 | 62 | 0.0123 | 0.2876 | 0.7124 | 0.0173 |
| 28 | 0.0052 | 0.0324 | 0.9676 | 0.0054 | 63 | 0.0117 | 0.5740 | 0.4260 | 0.0274 |
| 29 | 0.0023 | 0.0060 | 0.9940 | 0.0023 | 64 | 0.0125 | 0.3124 | 0.6876 | 0.0182 |
| 30 | 0.0019 | 0.0039 | 0.9961 | 0.0019 | 65 | 0.0059 | 0.8587 | 0.1413 | 0.0415 |
| 31 | 0.0027 | 0.0085 | 0.9915 | 0.0027 | 66 | 0.0106 | 0.6522 | 0.3478 | 0.0305 |
| 32 | 0.0023 | 0.0060 | 0.9940 | 0.0023 | 67 | 0.0126 | 0.4644 | 0.5356 | 0.0235 |

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|---|---|---|---|---|---|---|---|---|---|
| 33 | 0.0064 | 0.0498 | 0.9502 | 0.0067 | 68 | 0.0104 | 0.6627 | 0.3373 | 0.0310 |
| 34 | 0.0000 | 1.0000 | 0.0000 | 0.1129 | 69 | 0.0123 | 0.5018 | 0.4982 | 0.0248 |
| 35 | 0.0001 | 0.9984 | 0.0016 | 0.0755 | 70 | 0.0127 | 0.3629 | 0.6371 | 0.0199 |

Summing to up table which presents the probability density function, cumulative distribution function, reliability function and hazard function for all observations at different time.

We transform the real data above into the dummy variable based on the arithmetic mean which is calculated from the real failing time data (Mean = 46.53 min) as when the failing time below 46.53min takes zero otherwise takes one, and the transition frequency matrix has been calculated, then probability transition matrix computed as it shown below.

$$\text{number of transtions} = \begin{pmatrix} 32 & 7 \\ 6 & 25 \end{pmatrix}$$

$$P(x) = \begin{pmatrix} \frac{32}{39} & \frac{7}{39} \\ \frac{6}{31} & \frac{25}{31} \end{pmatrix}$$

$$P(x) = \begin{pmatrix} 0.8205 & 0.1795 \\ 0.1934 & 0.8066 \end{pmatrix}$$

The calculated mean value of (46.53) serves as a central measure of the system's average behavior. It is noteworthy that the probability of happening a failure less than 46.53 min and the followed failure is also less than the aforementioned value is 0.8205, happening a failure its time less than 46.53 min and the next failure takes more than the mentioned value its probability reaches 0.1795, happening a failure its time less than 46.53 min and the next failure takes less than the mentioned value its probability reaches 0.1934, the probability of happening a failure more than 46.53 min and the followed failure is also more than the aforementioned value is 0.8066.

The probabilities of transitioning from any state to any other state in two steps:

For predicting the probability transition matrix of the system for the next two years can be calculated by squaring the P(x).

$$P^2(x) = \begin{pmatrix} 0.6732 & 0.0322 \\ 0.0374 & 0.6506 \end{pmatrix}$$

The calculated mean value of (46.53) serves as a central measure of the system's average behavior. It is noteworthy that the probability of happening a failure less than 46.53 min and the followed failure is also less than the aforementioned value is 0.6732, happening a failure its time less than 46.53 min and the next failure takes more than the mentioned value its probability reaches 0.0322, happening a failure its time less than 46.53 min and the next failure takes less than the mentioned value its probability reaches 0.0374, the probability of happening a failure more than 46.53 min and the followed failure is also more than the aforementioned value is 0.6506. For prediction the probability transition matrix for long term time of the states (0,0) and (1,1) calculated as below:

$$(\pi_0 \ \ \pi_1)\begin{pmatrix} 0.8205 & 0.1795 \\ 0.1934 & 0.8066 \end{pmatrix} = \begin{pmatrix} \pi_0 \\ \pi_1 \end{pmatrix}$$

Setting up a system of equations to find the steady – state distribution vector $[\pi_0 \ \ \pi_1]$ for the Markov chain system of equation is:

$0.8205\ \pi_0 + 0.1934\ \pi_1 = \pi_0$ (3.1)

$0.1795\ \pi_0 + 0.8066\ \pi_1 = \pi_1$ (3.2)

$\pi_0 + \pi_1 = 1$ (3.3)

$\pi_0 = 0.5186$

$\pi_1 = 0.4814$

The derived steady-state distribution $[\pi_0 \ \ \pi_1] = [0.5186 \ \ 0.4814]$ the calculated mean value of (46.53) serves as a central measure of the system's average behavior. It is noteworthy that the probability of happening a failure less than 46.53 min and the followed failure is also less than the aforementioned value is 0.5186, the probability of happening a failure more than 46.53 min and the followed failure is also more than the aforementioned value is 0.4814.

**4-1. Conclusion:**

1. According to the figure (1) where the failure time at the beginning is occurred the most, this is because of the technician have no enough knowledge about how to use the machine we recommend who make concern to provide training for the staff.
2. From figure (3) the failure time reliability was stable from point (120).
3. The hazard function curve from figure (4) increased sharply and linearly.

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**4-2. Recommendations:** The calculated steady-state distribution vector [$\pi_0$, $\pi_1$] offers valuable insights into the probabilities associated with system failure times. Specifically, there is a 0.5186 probability that a failure will occur within 46.53 minutes and be followed by another failure. Conversely, there is a 0.4814 probability that a failure will occur after 46.53 minutes and be followed by another failure. These findings illuminate the temporal patterns of system failures, providing crucial information for understanding and predicting successive failures over time.

The laboratory must take precautions and develop a contingency plan based on the results in [$\pi_0$, $\pi_1$] to prevent and minimize machine malfunctions.

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