Tikrit Journal of Administrative and Economic Sciences, Vol. 21, No. 70, Part (1): 479-494
Doi: www.doi.org/10.25130/tjaes.21.70.1.25

Tikrit Journal of Administrative and Economics Sciences
مجلة تكريت للعلوم الإدارية والاقتصادية
EISSN: 3006-9149
PISSN: 1813-1719

Exponential Generalized Autoregressive Conditional Heteroscedastic Time Series Model Analysis with Wavelets

Hunar Adam Hamza*[A], Heyam A. A. Hayawi [B], Banaz Waleed Yaqoob Meran [A], Taha Hussein Ali [A]
[A] College of Administration and Economics / Salahaddin University / Erbil
[B] College Computer Science and Mathematics/ Mosul University

Keywords:
Time Series, EGARCH Model, Wavelet, Universal Threshold, Soft Rule.

ARTICLE INFO

Article history:

Received	11 Nov. 2024
Accepted	31 Dec. 2024
Available online	30 Jun. 2025

*Corresponding author:
Hunar Adam Hamza
College of Administration and Economics / Salahaddin University / Erbil

Abstract: Data noise is one of the problems facing the accuracy of building time series models, such as the EGARCH model. In this article, it was proposed to treat the noisy data of the Exponential Generalized Autoregressive Conditional Heteroscedastic Time Series Model using wavelet analysis through wavelets (Daubechies, Coiflets, and Symlets), with a Universal thresholding and the application of a soft threshold rule. The efficiency and accuracy of the estimated parameters of the Exponential Generalized Autoregressive Conditional Heteroscedastic model (for unprocessed data from noise) were compared with the three proposed models using the Akaike and Bayesian information criteria by studying simulation data and real data based on a program in the MATLAB language designed for this purpose. The research results demonstrated that the proposed methods were more efficient than the ordinary Exponential Generalized Autoregressive Conditional Heteroscedastic model.

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

تحليل نموذج السلاسل الزمنية المتغايرة الانحدارية الشرطية المعممة الأسية باستخدام الموجات

هونر آدم حمزة
كلية الادارة والاقتصاد
جامعة صلاح الدين-اربيل

هيام عبد المجيد حياوي
كلية علوم الحاسوب والرياضيات
جامعة الموصل

بناز وليد أيوب ميران
كلية الادارة والاقتصاد
جامعة صلاح الدين-اربيل

طه حسين علي
كلية الادارة والاقتصاد
جامعة صلاح الدين-اربيل

المستخلص

تعد ضوضاء البيانات إحدى المشكلات التي تواجه دقة بناء نماذج السلاسل الزمنية، مثل نموذج EGARCH. في هذه البحث، تم اقتراح معالجة البيانات من الضوضائية لنموذج السلاسل الزمنية غير المتجانسة المشروطة ذات الانحدار الذاتي العام الأسي باستخدام تحليل المويجات من خلال الموجات (Daubechies وCoiflets وSymlets)، مع تحديد قطع العتبة الشاملة وتطبيق قاعدة قطع العتبة الناعمة. تمت مقارنة كفاءة ودقة المعلمات المقدرة لنموذج الانحدار الذاتي العام الأسي (للبيانات غير المعالجة من الضوضاء) مع النماذج الثلاث المقترحة باستخدام معايير المعلومات Akaike وBayesian من خلال دراسة بيانات المحاكاة والبيانات الحقيقية بناءً على برنامج بلغة MATLAB مصمم لهذا الغرض. أظهرت نتائج البحث أن الطرق المقترحة كانت أكثر كفاءة من نموذج الانحدار الذاتي العام الأسي الاعتيادي.

الكلمات المفتاحية: السلاسل الزمنية، نموذج EGARCH، المويجة، قطع العتبة الشاملة والقاعدة الناعمة.

1. Introduction

A time series is a chronologically ordered set of observations, the time intervals of which are often equal and successive, and these intervals vary depending on the nature of the geographical phenomenon (Ali et al., 2024: 352). Time series are used in several areas, including spatial studies, whether natural, population or economic.; Where time series models are usually used to predict the values of a variable such as: predicting the state of the atmosphere, temperatures, and growth in education (Mustafa and Ali, 2013:193). Models are used if the variable to be studied has unknown determinants, or the factors influencing it (Ali et al., 2024: 1374). It is also used in the case that the variable is subject to the expectations of those dealing with it, which is reflected in the expectation of the future based on what happened in the past, as the variable, like other variables, moves with time, we expect it to take several forms and directions when studied, such as being in a linear form, or a non-linear form, and the case of direct or inverse correlation weak or strong. Several factors affect the development of the variable according to time, leading to its increase or decrease, and the most important of these factors are the so-called components of the time series (Ali et al., 2022: 435). Reducing noise in time series data is an important issue. Wavelet analysis is a new powerful tool for financial time series analysis and reduces the noise of the data (de-noise). Wavelets can decompose a signal or a time series on different levels. Therefore, this decomposition reveals the structure of the underlying signal and trends, periodicities jump, or singularities (Dautov and Özerdem, 2018: 3).

The EGARCH model has applications in many fields, including systematic risk analysis, (Blazsek et al., 2018: 6052) introduce new Markov-switching dynamic conditional score EGARCH models, to be used by practitioners for forecasting value-at-risk and expected shortfall in systematic risk analysis. Also, in finance markets, where the researcher (Villar et al., 2023: 505) applied this model to predict volatility in unconsolidated financial markets, the case of European carbon allowances.

In this article, it was proposed to treat the noisy data of the Exponential Generalized Autoregressive Conditional Heteroscedastic Time Series Model using wavelet analysis through wavelets (Daubechies, Coiflets, and Symlets) of different orders (4, 3 and 2), respectively, with a Universal thresholding and the application of a soft threshold rule. The theoretical aspect included an introduction to the EGARCH model, wavelets, and their types used in the practical aspect, with estimating the thresholding and the rule adopted in the threshold, in addition to the efficiency criteria of the estimated model, and proposed method, while the practical aspect dealt with simulating the model, in addition to real data.

2. **Time Series:** A time series is a set of interrelated observations of a phenomenon indexed according to its acquired rank in time and these observations are recorded sequentially and separated by equal periods in time or close to equal, for example, an hour, a day, a week, a month or a year, and so on, and usually the observations are not independent of each other and this property is used in the formation of a time series model, or a phrase that depends on the nature of the phenomenon being studied to establish a scientific assumption (Shahla et al., 2023: 139).
3. **EGARCH Model:** The model which stands for Exponential Generalized Autoregressive Conditional Heteroscedasticity, is a model used in econometrics and finance to describe time series data with volatility

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

clustering. It extends the GARCH (Generalized Autoregressive Conditional Heteroscedasticity) model by allowing for asymmetric effects of positive and negative shocks and modelling leverage effects. An EGARCH (p, q) model can be expressed as:

$$\ln\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i * y_{t-i} + \gamma_j|(y_{t-i}| - E|y_{t-i}|)) + \sum_{i=1}^{q}\beta_i * \ln\sigma_{t-i}^2 \quad (1)$$

where the coefficient α_i captures the sign effect and γ_j the size effect. The exponential operator to get conditional volatility guarantees its positivity, so no restrictions are required, for conditions' volatility is positive (Chang and McAleer, 2017: 54). The great advantage of the EGARCH model is that it allows us to reflect on how positive and negative changes in the series affect volatility, which is not the case in the GARCH model. The conditional variance of the GARCH model is a function of the square of the past innovations, so it does not collect positive or negative changes (Dalya & Mundher, 2024: 392).

4. **Wavelets:** Small waves, called wavelets, can be combined to create larger or different waves. To create a wavelet system that could accurately copycat any wave, a few basic waves were employed and then extended and adjusted in an endless variety of ways (Halidou et al., 2023: 41555). Consider making an orthogonal wavelet, the basis for the functions $f \in L^2(R)$ (the space of squarely integration actual functions), opening with two parent wavelets: the scaling function (also called the farther, wavelet ϕ) and the mother wavelet (ψ). Other wavelets are then created by dilations and translations of (ϕ) and (ψ) (Percival et al., 2004, p.78). Formulas define the dilation and translation of the functions. (1 and 2).

$$\phi_{k,q}(y) = 2^{\frac{k}{2}}\phi(2^k y - q) \quad k, q \in z \qquad (2)$$

$$\psi_{k,q}(y) = 2^{\frac{k}{2}}\psi(2^k y - q) \quad k, q \in z \qquad (3)$$

4-1. Daubechies Wavelet: Daubechies Wavelet: The Daubechies wavelet is a family of orthogonal wavelets used in signal processing and data de-noising, named for the Belgian mathematician Ingrid Daubechies. These wavelets are well-liked for a variety of applications due to their orthogonality, symmetry, and compact support (Wen et al., 2022: 65).
Daubechies wavelets have several important characteristics.

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

1. Orthogonality: Daubechies wavelets protect energy, and information through transformation by forming an orthogonal basis for data decomposition.
2. Compact Support: Daubechies wavelets have finite support, which indicates that they are non-zero throughout a finite interval, in contrast to several other wavelet families. This quality is beneficial for the study of confined signals (Omer et al., 2024: 447).
3. Variability: The Daubechies wavelet family consists of several members, each with a distinct number of vanishing moments, including Daubechies D2, D3, D4, D5, and so forth, (Omar et al., 2024: 116).

This decomposition makes it calmer to extract features, denoise, compress, and analyze signals. Several domains, including image and signal processing, de-noising data, feature extraction, and pattern, identification, use Daubechies wavelets. Their characteristics make them ideal for occupations requiring effectively depicting localized features in data, (Wang et al., 2018: 122). Asymmetric; lacks symmetry, which may result in slight distortions when the data is reconstructed.

4-2. The Coiflets: There are several areas where Coiflets, and wavelets, have been demonstrated to be useful and proved their utility, in the method of moments, namely in 3-D scattering from rough surfaces, by lowering the sampling rate and compressing, the matrix size, (Pan et al., 2004: 3099). They suggested the generalized Coiflets, a new family of orthonormal wavelets with enhanced properties like near-symmetry and near-linear phase, (Wei et al., 1997: 1260). Daubechies and Coiflets are built following specific, integer selections of the shift α. There are several more Coiflets instances in the literature that are still for integer shifts, (Monzón et al., 1999: 192).

Coiflets have an equal number of zero moments in the scaling function and the analytic wavelet function. This enhances their ability to analyze data that contain low-frequency trends while preserving fine details. Compared to Daubechies wavelets, Coiflets have improved (nearly) analogue properties, reducing distortions in the resulting signal when it is reconstructed (signal reconstruction).

4-3. Symlets: Wavelet Symlets, N is the order in Sym(N). Some authors substitute 2N for N (Ali et al., 2023: 15). As additions to the DB family, Daubechies presented the nearly symmetrical, orthogonal, and biorthogonal wavelets, known as Symlets. The two wavelet families' characteristics are comparable (Chavan et al., 2011, p.145). It shares with Daubechies and Coiflets the fact that it is an orthogonal wavelet and supports multi-resolution analysis. However, it has features that distinguish it from other wavelets such as Daubechies, Coiflets and Haar:

5. **Soft Threshold Rule**: Soft thresholding is used in signal and image processing or de-noise data, especially with wavelet shrinkage. It entails decreasing the wavelet coefficients' magnitude by a convincing amount without precisely setting them to zero. The definition of the soft thresholding function is as follows (Lei et al., 2021:78):

$$Wn^{(st)} = \mathrm{sign}\{Wn\}(|Wn| - l)_{+} \qquad (4)$$

where l stands for the value at the threshold (Ali et al., 2022: 20) and

$$\mathrm{Sign}\{Wn\} = \begin{bmatrix} +1 & \text{if } Wn > 0 \\ -1 & \text{if } Wn = 0 \\ 0 & \text{if } Wn < 0 \end{bmatrix} \qquad (5)$$

and

$$(|Wn| - l)_{+} = \begin{bmatrix} (|Wn| - l) & \text{if } (|Wn| - l) \geq 0 \\ 0 & \text{if } (|Wn| - l) < 0 \end{bmatrix} \qquad (6)$$

6. **Information criterion:** Some statistical criteria can be used to compare the estimated models' efficiency.

6-1. The Akaike Information Criterion (AIC): As a model selection criterion for evaluating real data, the Akaike Information Criterion has been instrumental in resolving problems across numerous domains. The model with the lowest AIC is optimal (Akaike, 1974).

$$AIC = -2 \,\mathrm{Ln}\, L + 2K \qquad (7)$$

Where K represents the number of Parameters in the model and L is the maximum value of the likelihood function for the model.

6-2. The Bayesian information criterion (BIC): The Bayesian information criterion is one of the most effective, well-known, and widely useful methods in statistical model selection. Each model's BIC is determined, and the model that has the lowest BIC value is preferred as the best model (Schwarz, 1978).

$$BIC = -2 \,\mathrm{Ln}\, L + K \,\mathrm{Ln}\, N \qquad (8)$$

Where N: represents the number of Observations.

7. **Proposed Method:** The proposed method relies on processing the data of the EGARCH model from noise before estimating its parameters using wavelet shrinking, which relies on the following steps:

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

- ❖ Using the wavelets (Daubechies, Coiflets, and Symlets) of different orders with time series data to obtain maximal overlap discrete wavelet transformation coefficients.
- ❖ Estimate the thresholding level (η) using the universal threshold from the first level for Maximal overlap discrete wavelet transformation coefficients (Haydier et al., 2023: 254).
- ❖ Use the soft threshold rule on the maximal overlap discrete wavelet transformation coefficients at the estimated threshold level (η) by keeping or killing (converting them to zero) to obtain modified coefficients with little noise.
- ❖ Finding the inverse of the maximal overlap discrete wavelet transformation coefficients for de-noise data while preserving 99.97% of the data energy.
- ❖ Using de-noise time series data to estimate EGARCH model parameters.

8. Simulation Study: This section examines the effectiveness of wavelet denoising for improving EGARCH model performance. This study used wavelets (Daubechies, Coiflets, and Symlets) with different orders (4, 3 and 2) for data denoising. A universal threshold type and a soft threshold rule were applied to reduce noise in the time series data. Then, filtered data sets were generated for EGARCH model estimation. The models were estimated using a sample size of n = 78.

Several different wavelets were used to process the problem of data noise and what was mentioned in the article is only the wavelets which are suitable for the EGARCH model. Figure 1 shows the original time series observations and the results of the wavelet shrinkage process that provided de-noise data for Db4. The steps suggested in paragraph (7) were applied and the model that gave the best results based on the criteria AIC and BIC is the best and has the least noise due to using the same models before and after wavelet shrinkage.

To assess model performance, Statistical criteria, including the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), were used to evaluate the performance of each model based on its forecast accuracy. The results of this evaluation are summarized in Table 1 and Figure (2 to Figure (5.

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

Table (1): Estimated Parameter with forecast accuracy

Models	Stat	Value	Standard Error	t-Statistic	p-value	AIC	BIC
EGARCH	Constant(ω)	-5.352	0.680	-7.875	0.000	-17.71	-5.92
	GARCH(α)	-1.000	0.099	-10.072	0.000		
	ARCH(β)	0.488	0.355	1.374	0.169		
	Leverage(γ)	-0.091	0.244	-0.373	0.709		
	Offset(δ)	0.092	0.037	2.521	0.012		
EGARCH with WAVELET DB (4)	Constant(ω)	-1.168	0.517	-2.259	0.024	-265.97	-254.19
	GARCH(α)	0.823	0.084	9.797	0.000		
	ARCH(β)	1.000	0.268	3.726	0.000		
	Leverage(γ)	0.463	0.199	2.333	0.020		
	Offset(δ)	0.089	0.004	22.708	0.000		
EGARCH with WAVELET DB (3)	Constant(ω)	-4.556	1.194	-3.815	0.000	-199.99	-188.20
	GARCH(α)	0.197	0.197	0.998	0.318		
	ARCH(β)	1.000	0.342	2.921	0.003		
	Leverage(γ)	0.167	0.247	0.674	0.500		
	Offset(δ)	0.094	0.007	13.850	0.000		
EGARCH with WAVELET DB (2)	Constant(ω)	-6.073	1.405	-4.321	0.000	-184.55	-172.77
	GARCH(α)	-0.113	0.276	-0.410	0.682		
	ARCH(β)	1.000	0.391	2.556	0.011		
	Leverage(γ)	-0.216	0.296	-0.729	0.466		
	Offset(δ)	0.127	0.009	14.560	0.000		
EGARCH with WAVELET Coiflets (4)	Constant(ω)	-1.29	0.46	-2.81	0.005	-291.71	-279.93
	GARCH(α)	0.82	0.07	11.45	0.000		
	ARCH(β)	1.00	0.21	4.88	0.000		
	Leverage(γ)	0.45	0.17	2.68	0.007		
	Offset(δ)	0.09	0.00	31.53	0.000		
EGARCH with WAVELET Coiflets (3)	Constant(ω)	-1.17	0.59	-1.98	0.047	-243.17	-231.38
	GARCH(α)	0.82	0.10	8.21	0.000		
	ARCH(β)	1.00	0.28	3.57	0.000		
	Leverage(γ)	0.24	0.23	1.08	0.278		
	Offset(δ)	0.09	0.00	19.49	0.000		

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

Models	Stat	Value	Standard Error	t-Statistic	p-value	AIC	BIC
EGARCH with WAVELET Coiflets (2)	Constant(ω)	-0.87	0.41	-2.12	0.034	-281.52	-269.74
	GARCH(α)	0.88	0.07	13.00	0.000		
	ARCH(β)	1.00	0.19	5.29	0.000		
	Leverage(γ)	0.47	0.16	3.01	0.003		
	Offset(δ)	0.09	0.00	38.56	0.000		
EGARCH with WAVELET Symlets (4)	Constant(ω)	-1.55	0.63	-2.45	0.014	-255.00	-243.22
	GARCH(α)	0.78	0.09	8.24	0.000		
	ARCH(β)	1.00	0.24	4.25	0.000		
	Leverage(γ)	0.27	0.21	1.30	0.194		
	Offset(δ)	0.08	0.00	22.65	0.000		
EGARCH with WAVELET Symlets (3)	Constant(ω)	-4.56	1.19	-3.81	0.000	-199.99	-188.20
	GARCH(α)	0.20	0.20	1.00	0.318		
	ARCH(β)	1.00	0.34	2.92	0.003		
	Leverage(γ)	0.17	0.25	0.67	0.500		
	Offset(δ)	0.09	0.01	13.85	0.000		
EGARCH with WAVELET Symlets (2)	Constant(ω)	-6.07	1.41	-4.32	0.000	-184.55	-172.77
	GARCH(α)	-0.11	0.28	-0.41	0.682		
	ARCH(β)	1.00	0.39	2.56	0.011		
	Leverage(γ)	-0.22	0.30	-0.73	0.466		
	Offset(δ)	0.13	0.01	14.56	0.000		

The analysis of Table 1 strongly suggests that the proposed method using wavelet denoising for data pre-processing leads to more accurate and informative parameter estimates in the EGARCH model. The denoising process helps reveal underlying relationships in the data, leading to statistically significant leverage terms and a better representation of the volatility dynamics. Additionally, the substantial improvement in AIC and BIC values across both model specifications indicates a superior model fit with reduced complexity for the proposed method.

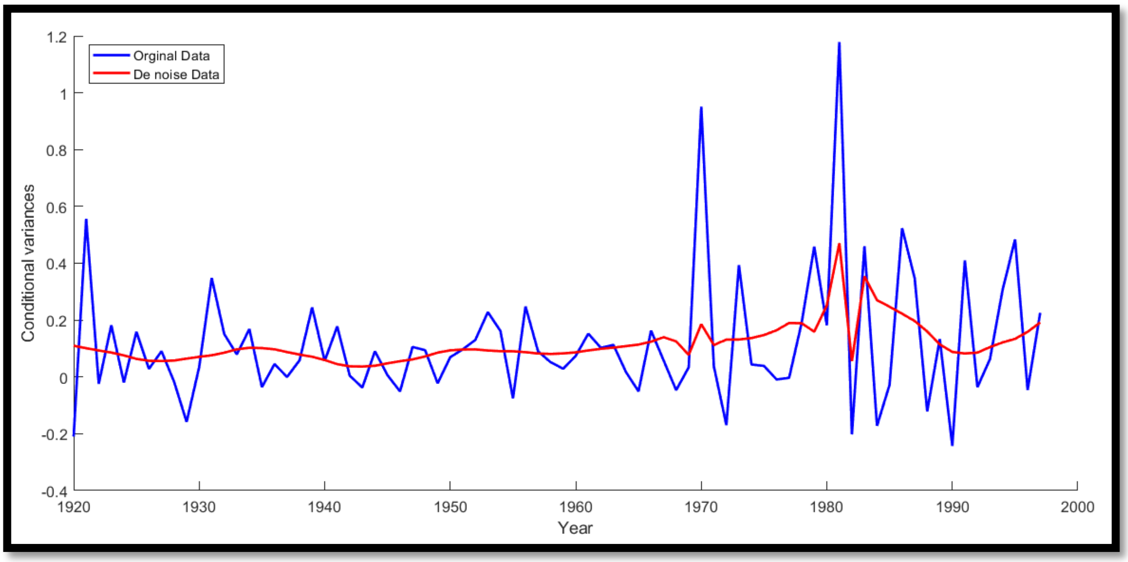

Figure (1) Denoise Data using Db4

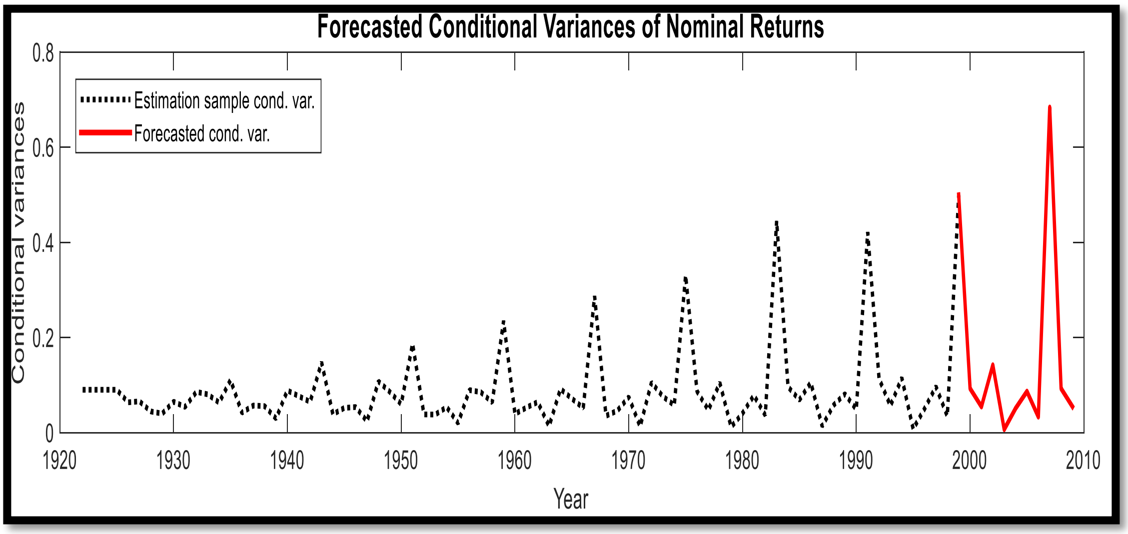

Figure (2): EGARCH model without Wavelet

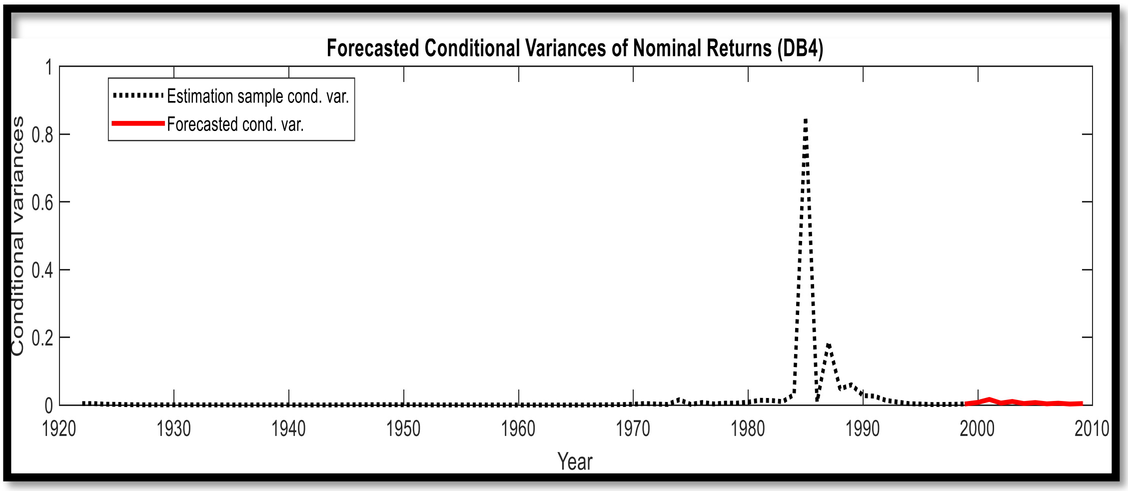

Figure (3): EGARCH model with Wavelet (DB4)

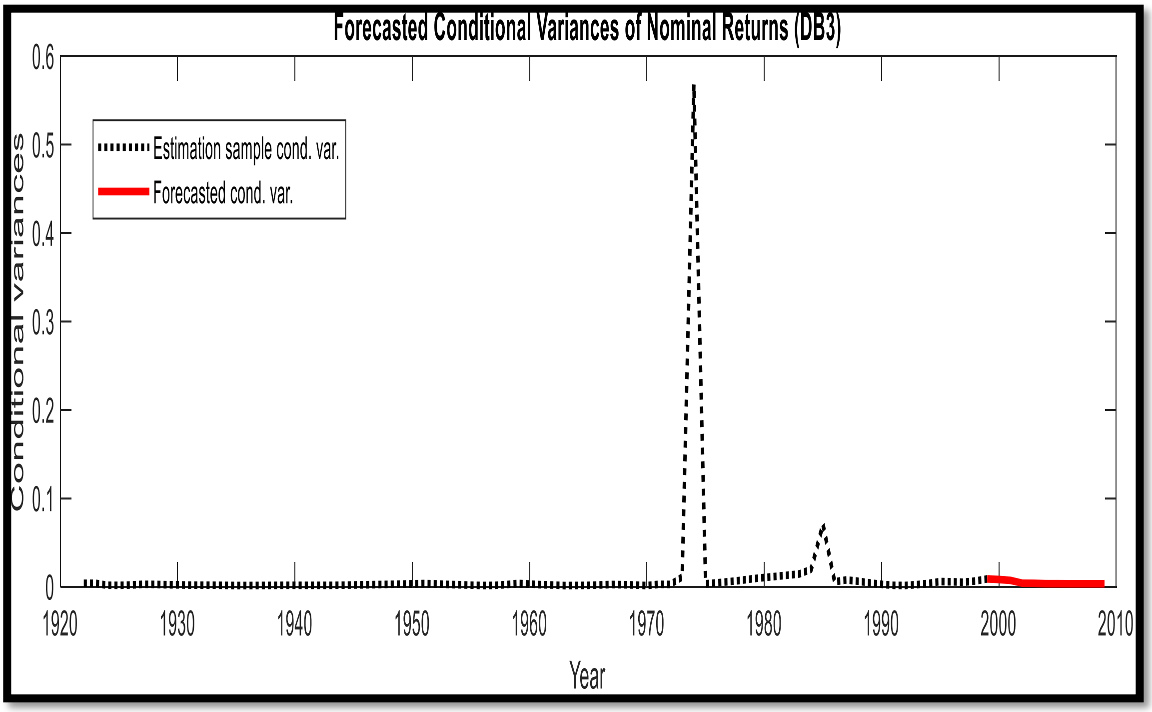

Figure (4): EGARCH model with Wavelet (DB3)

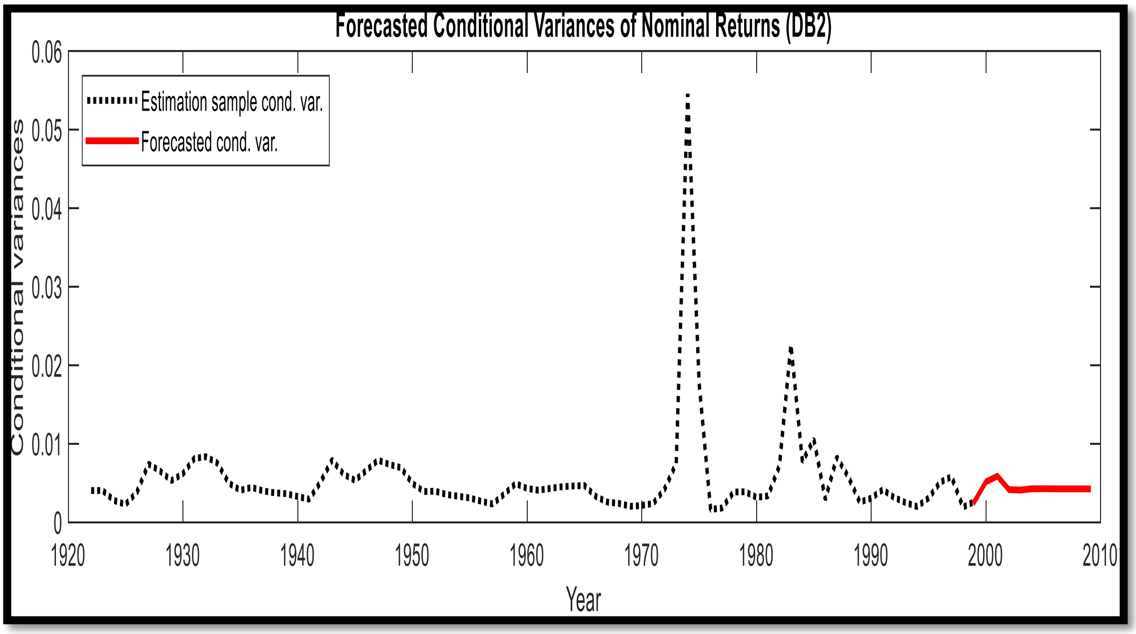

Figure (5): EGARCH model with Wavelet (DB2)

A Monte Carlo simulation was conducted to evaluate the proposed model's performance across various simulated scenarios. The simulation involved 1000 replications, where each replication generated a new data set. For each replication, the proposed model was fitted, and the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were calculated. The results of these calculations are summarised in Table 2.

Table (2): Information Criterion Comparison

WAVELET Type	Information Criterion	Order		
		2	3	4
Daubechies	AIC	-264.3	-285.3	-294.8
	BIC	-247.8	-268.8	-274.3
Coiflets	AIC	-286.6	-307.1	-311.5
	BIC	-270.1	-290.6	-295.0
Symlets	AIC	-264.3	-285.3	-283.5
	BIC	-247.8	-268.8	-267.0
Classical Method	AIC	-17.71	-17.71	-17.71
	BIC	-5.92	-5.92	-5.92

The results from the simulation show that wavelet denoising techniques can significantly improve the performance of the EGARCH model, as measured by AIC and BIC in Table (1). Among the wavelet types explored, Coiflets seem to achieve the lowest AIC and BIC values in most cases, indicating potentially better performance in denoising for this specific data set (Danish Data) and model combination (EGARCH with Wavelet). This suggests that denoising the data with wavelets generally improves the model's ability to capture the underlying structure of the data while avoiding unnecessary complexity.

9. **Real Data:** This study uses nominal returns (RN) extracted from an annual time series of Danish nominal stock returns spanning the period 1922 to 1999. The data source for this information is the study titled 'Stock Returns and Bond Yields in Denmark, 1922-1999' by (Nielsen & Risager, 2001, p.78). A visual representation of the nominal returns (RN) is presented in Figure (6.

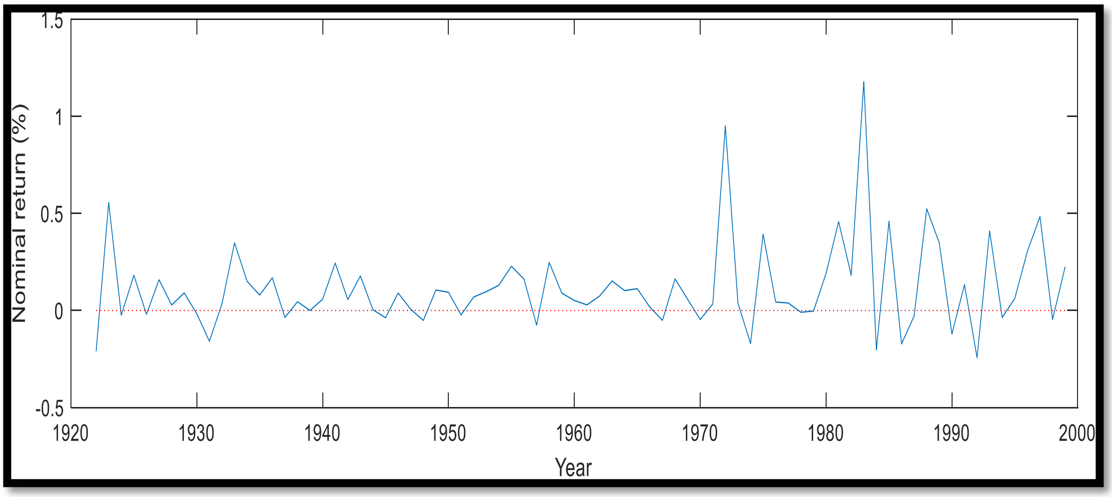

Figure (6): Nominal returns (RN) of Danish stock from 1922 to 1999

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

A Daubechies wavelet function (DB) with orders of 4, 3, and 2 was used for denoising. The results of this denoising process are visually presented in Figure (7 to Figure (9.

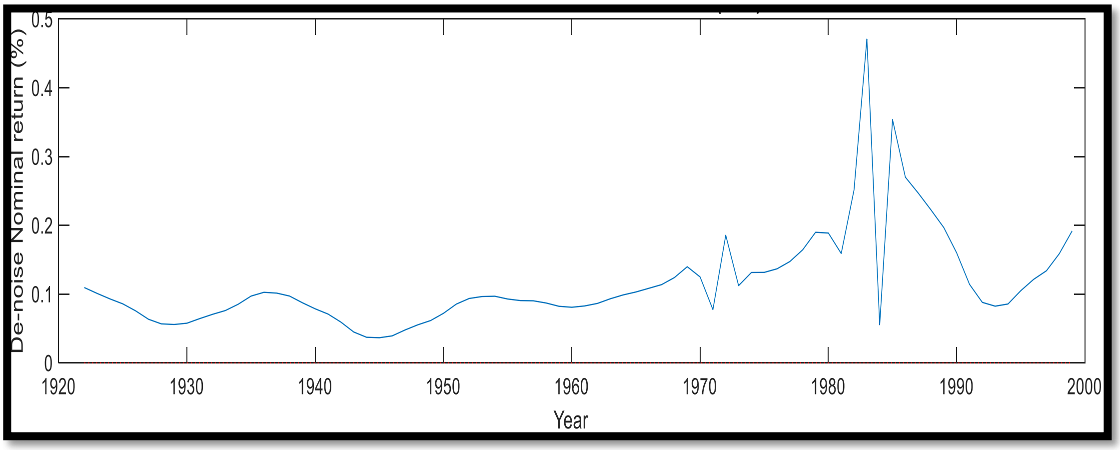

Figure (7): De-noise Danish Nominal Stock Returns (DB4)

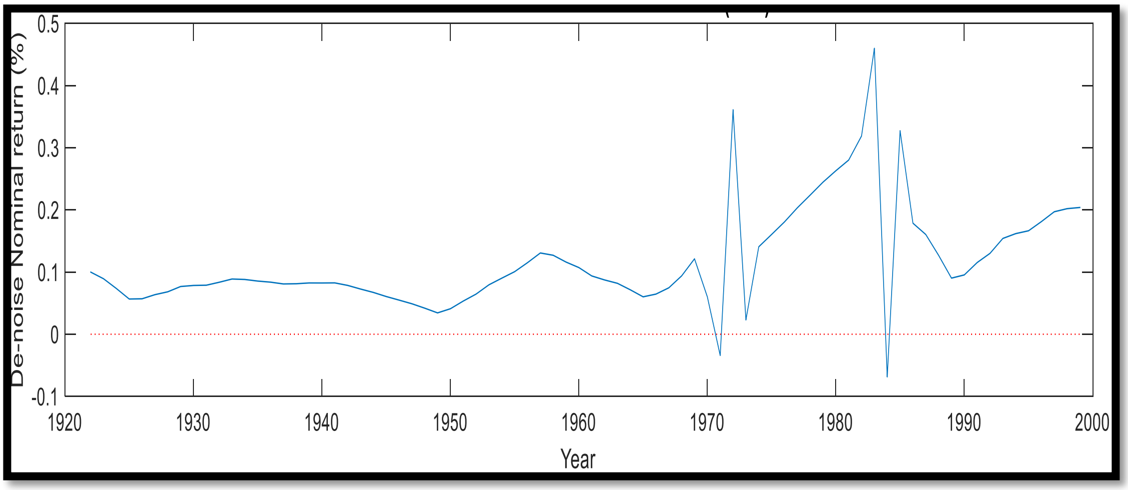

Figure (8): De-noise Danish Nominal Stock Returns (DB3)

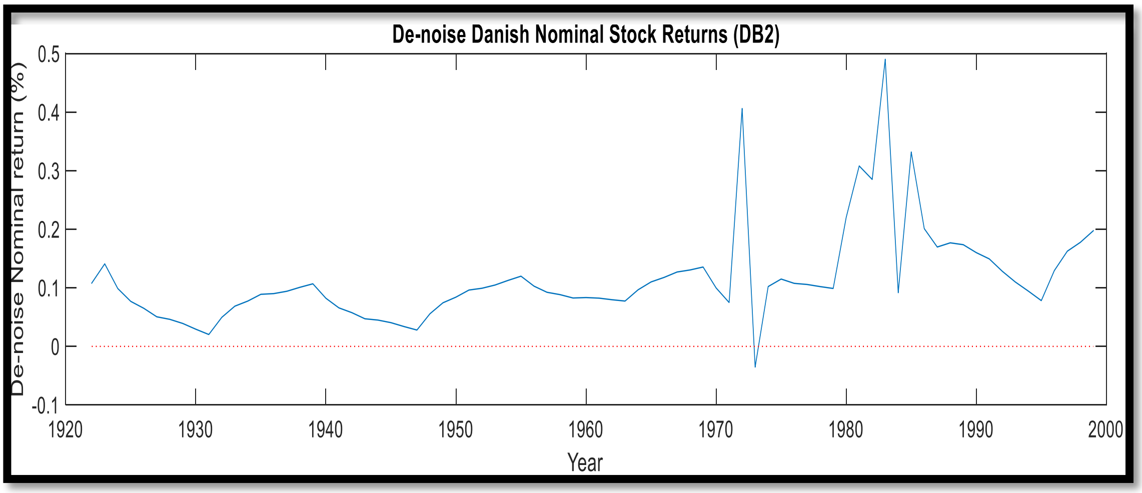

Figure (9) De-noise Danish Nominal Stock Returns (DB2)

A good time series model will have an ACF plot for residuals with no significant spikes (all values within the confidence intervals), and the model that has the least AIC and BIC is the most accurate.

10. Conclusions: Overall, the findings strongly support the idea that wavelet denoising techniques can be a valuable pre-processing step for EGARCH models, particularly using Coiflets wavelets. By removing noise from the data, wavelets allow the model to focus on the underlying structure, leading to more accurate estimates of the parameters that rule volatility dynamics and potentially better forecasting performance.

11. Recommendations: Based on the findings of this research, we recommend to:

1. Wavelet denoising can be used as a pre-processing step for EGARCH models, as the wavelet techniques effectively remove noise from the data.
2. Use and explore a varied range of wavelet functions and develop data-driven methods for selecting the optimal wavelet and order for a specific data set
3. Use forecasting accuracy such as AIC and BIC to choose the best model fit.

References

1. Akaike, H. (1974). A new look at the statistical model identification. IEEE Transactions on Automatic Control, 19(6), 716–723.
2. Ali, T. H., Mahmood, S. H., & Wahdi, A. S. (2022). Using Proposed Hybrid method for neural networks and wavelet to estimate time series model. Tikrit Journal of Administrative and Economic Sciences, 18(57, 3), 432–448. https://doi.org/10.25130/tjaes.18.57.3.26
3. Ali, T. H., Raza, M. S., & Abdulqader, Q. M. (2024). VAR TIME SERIES ANALYSIS USING WAVELET SHRINKAGE WITH APPLICATION. Science Journal of University of Zakho, 12(3), 345–355. https://doi.org/10.25271/sjuoz.2024.12.3.1304
4. Ali, T. H., Sedeeq, B. S., Saleh, D. M., & Rahim, A. G. (2024). Robust multivariate quality control charts for enhanced variability monitoring. Quality and Reliability Engineering International, 40(3), 1369-1381. https://doi.org/10.1002/qre.3472
5. Ali, Taha Hussein and Jwana Rostam Qadir, (2022) "Using Wavelet Shrinkage in the Cox Proportional Hazards Regression model (simulation study)", Iraqi Journal of Statistical Sciences, 19, 1: 17-29.
6. Ali, Taha Hussein, Nasradeen Haj Salih Albarwari, and Diyar Lazgeen Ramadhan, (2023). “Using the hybrid proposed method for Quantile Regression and Multivariate Wavelet in estimating the linear model parameters.” Iraqi Journal of Statistical Sciences 20.1: 9-24.
7. Blazsek, S., Ho, H. C., & Liu, S. P. (2018). Score-driven Markov-switching EGARCH models: an application to systematic risk analysis. Applied Economics, 50(56), 6047–6060. https://doi.org/10.1080/00036846.2018.1488073

8. Chang, C., & McAleer, M. (2017). The correct regularity condition and interpretation of asymmetry in EGARCH. Economics Letters, 161, 52-55. https://doi.org/10.1016/j.econlet.2017.09.017
9. 9. Chavan, M. S., Mastorakis, N., Chavan, M. N. &Gaikwad, M. (2011). Implementation of SYMLET wavelets to remove Gaussian additive noise from the speech signal. 10th WSEAS International Conference on Electronics, Hardware, Wireless Optical Communications (EHAC’11), Cambridge, 37
10. 10. Dalya Dh. Ahmed, & Mundher A. Khaleel. (2024). The Gompertz Inverted Nadarajah-Haghighi (GoINH) Distribution Properties with Application to Real Data. Tikrit Journal of Administrative and Economic Sciences, 20(67, part 2), 386–402. https://doi.org/10.25130/tjaes.20.67.2.21
11. 11. Dautov Ç. P. and Özerdem M. S., (2018). "Wavelet transform and signal denoising using Wavelet method," 2018 26th Signal Processing and Communications Applications Conference (SIU), Izmir, Turkey, pp. 1-4, doi: 10.1109/SIU.2018.8404418.
12. 12. Halidou, A., Mohamadou, Y., Ari, A. A. A. et al., (2023). Review of wavelet denoising algorithms. Multimed Tools Appl 82, 41539–41569 (2023). https://doi.org/10.1007/s11042-023-15127-0
13. 13. Haydier, Esraa Awni, Nasradeen Haj Salih Albarwari, and Ali, Taha Hussein, (2023). "The Comparison Between VAR and ARIMAX Time Series Models in Forecasting." Iraqi Journal of Statistical Sciences 20(2), 249-262.
14. 14. Lei, S., Lu, M., Lin, J. et al., (2021). Remote sensing image denoising based on improved semi-soft threshold. SIViP 15, 73–81. https://doi.org/10.1007/s11760-020-01722-3
15. 15. Monzón, L., Beylkin, G., & Hereman, W. (1999). Compactly supported wavelets based on almost interpolating and nearly linear phase filters (coiflets). Applied and Computational Harmonic Analysis, 7(2), 184-210.
16. 16. Mustafa, Qais and Ali, Taha Hussein, (2013). "Comparing the Box Jenkins models before and after the wavelet filtering in terms of reducing the orders with application." Journal of Concrete and Applicable Mathematics 11: 190-198.
17. 17. Nielsen, S., & Risager, O. (2001). Stock returns and bond yields in Denmark, 1922–1999. Scandinavian Economic History Review, 49(1), 63–82. https://doi.org/10.1080/03585522.2001.10419841
18. 18. Omar Ramzi Jasim, & Sarmad Abdulkhaleq Salih. (2024). Comparison between the Bayesian method and the Particle Swarm Algorithm for Estimating a Spatial Autoregressive Model. Tikrit Journal of Administrative and Economic Sciences, 20(67, part 1), 443–456. https://doi.org/10.25130/tjaes.20.67.1.22
19. 19. Omer, A. W., Sedeeq, B. S., & Ali, T. H. (2024). A proposed hybrid method for Multivariate Linear Regression Model and Multivariate Wavelets (Simulation study). Polytechnic Journal of Humanities and Social Sciences, 5(1), 112-124.
20. 20. Pan, G., Wang, K., & Cochran, D. (2004). Coifman wavelets in 3-D scattering from very rough random surfaces. IEEE Transactions on Antennas and Propagation, 52, 3096-3103.

Doi: www.doi.org/10.25130/tjaes.21.70.1.25

21. 21. Percival, D. B., Wang, M., & Overland, J. E. (2004), An introduction to wavelet analysis with applications to vegetation time series. Community Ecology.
22. 22. Schwarz, G. (1978). Estimating the Dimension of a Model. The Annals of Statistics, 6(2), 461–464.
23. 23. Shahla Hani Ali, Heyam A.A.Hayawi, Nazeera Sedeek K., and Taha Hussein Ali, (2023) "Predicting the Consumer price index and inflation average for the Kurdistan Region of Iraq using a dynamic model of neural networks with time series", The 7th International Conference of Union if Arab Statistician-Cairo, Egypt 8-9/3/2023:137-147.
24. 24. Villar-Rubio, E., Huete-Morales, MD. & Galán-Valdivieso, F. Using EGARCH models to predict volatility in unconsolidated financial markets: the case of European carbon allowances. J Environ Stud Sci 13, 500–509 (2023). https://doi.org/10.1007/s13412-023-00838-5
25. 25. Wang, SH., Zhang, YD., Dong, Z., Phillips, P. (2018). Wavelet Families and Variants. In: Pathological Brain Detection. Brain Informatics and Health. Springer, Singapore. https://doi.org/10.1007/978-981-10-4026-9_6
26. 26. Wei, D., Bovik, A.C., & Evans, B.L. (1997). Generalized Coiflets: a new family of orthonormal wavelets. Conference Record of the Thirty-First Asilomar Conference on Signals, Systems and Computers (Cat. No.97CB36136), 2, 1259-1263 vol.2.
27. 27. Wen, Q., Zhou, T., Zhang, C., Chen, W., Ma, Z., Yan, J., & Sun, L. (2022). Transformers in Time Series: A Survey. ArXiv. /abs/2202.07125.