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[0,1]Truncated Exponential Marshall Olkin Rayleigh distribution Properties and Applications

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Abstract:

In this research we suggest a new compound distribution called [0,1]Truncated Exponential Marshall Olkin Rayleigh (TEMO-R). This distribution has three parameters therefore it has very flexibility for fitting many different types of data. This distribution gives different shapes such as shape symmetrical, right-skewed. The probability density function has been expanded and many mathematical and statistical characteristics have been created, were found including the Quantile Function, the moments, the moments - generating function, entropy, order statistics. By maximum likelihood method was estimated the parameters of the new distribution.

Keywords: new distribution, Moment, Renyi Entropy, Order statistics, Maximum likelihood method.

[0, 1] توزيع الأسي مارشال اولكين رالي الاسي المبتور الخصائص مع التطبيقات

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المستخلص:

في هذا البحث نقترح توزيعًا جديدًا يسمى:
[0,1]Truncated Exponential Marshall Olkin Rayleigh (TEMO-R).
يحتوي هذا التوزيع على ثلاث معلمات ما يجعل التوزيع يتمتع بمرونة عالية في التعامل مع انواع مختلفة من البيانات. يعطي هذا التوزيع أشكالًا مختلفة مثل الشكل المتطابق أو الملتوي إلى اليمين. تمت إعادة صياغة دالة كثافة الاحتمال وتم إنشاء العديد من الخصائص الرياضية والإحصائية، بما في ذلك الدالة الكمية، العزوم، الدالة المولدة للعزوم، والإنتروبيا، الاحصائيات المرتبة. بواسطة طريقة الامكان الاعظم تم تقدير معلمات التوزيع الجديد.

الكلمات المفتاحية: توزيع جديد، العزوم، انتروبيا ريني، احصاءات مرتبة، نظرية الإمكان الأعظم.

1. Introduction:

Probability distribution has many applications in describing real-world situations. A lot of research has shown that some real-life data that cannot be properly modeled through traditional statistical distributions, so many researchers have resorted to finding new distributions by adding one, two or three parameters. (marshall, and Olkin, 1997)

They Suggested a new family by adding a shape parameter to the basic distribution. (Al-Saiari et al., 2014) using the Marshall Olkin family by expanded the distribution of Burr Xıı which was applied to electrostatic insulation data. (Yousof et al., 2018) using the Marshall Olkin family by expanded Weibull 's distribution and applied it to cancer patients data. (AHmad, H. H., & Almetwally, E. 2020) By circulating the distribution of Paret using the Marshall Olkin distribution this distribution was applied to engineering data. (Eugene et al., 2002) proposed a new family and called it Beta-G based on Logic of a Beta by adding two shape parameters to the basic distribution. (Cordeiro et al., 2011)

Suggested a new family called Kumaraswamy-G where they used kumaraswamy distribution over the period [0.1] to add two shape parameters to the basic distribution. (Gupta et al., 1998) suggested to generalize a new exponential-G family by adding a single shape parameter to the basic distribution. (Zografos, and Balakrishnan, 2009)

Proposed the Gamma-G-type 1 family to add a form parameter to the basic distribution, where a range of continuous single distributions were studied to generalize gamma distribution and codify theoretical values. (ristic, and Balakrishnan, 2012) researchers studied The gamma-exponentiated exponential distribution.The Rayleigh's distribution was first introduced by Lord Rayleigh in 1880 and was found as a result of the acoustics problem. And it's appeared as special case of the Weibull distribution. This distribution is very popular among lifelong distributions, as well as in researchers have studied this distribution by deriving some of its mathematical characteristics and its relationship with some families and other distributions, for example, lindley-Rayleigh distribution (Cakmakyapan, and Özel, 2018).

Extended Reciprocal Rayleigh Distribution (Rezk H. 2020). Odd generalized exponential Rayleigh distribution (Luguterah, A. 2016). In this research, a new model has been found. TEMO-R and prove that its flexibility is more true data than the rest of the distributions. The second section ensures the finding of the cumulative distributional function CDF, the probability density function PDF and And the risk function HF and the third section to expand the probability density function PDF and the cumulative function. The fourth section includes finding and studying some important mathematical characteristics such as moment ,moment generating function, quantity function, ordered statistics, and entropy. In the Section five the parameters estimate was taken For the new distribution using the Maximum likelihood method, the sixth section is about the application. In the end Conclusions and References.

2. [0,1]Truncated Exponential Marshall Olkin Rayleigh distribution (TEMO-R):

Let's have Rayleigh distribution , the CDF and the PDF of Rayleigh distribution is given as:

$$G(x,\beta) = 1 - e^{-(\beta x)^2} \qquad \beta > 0, \quad x > 0 \tag{1}$$

$$g(x,\beta) = 2\,\beta^2 x e^{-(\beta x)^2} \tag{2}$$

To find the [0,1] Truncated Exponential Marshall-Olkin-G family we will follow the method by (Abid, & Abdulrazak, 2017).

The CDF and PDF of new family are given as:

$$F(x;\alpha,\theta,\beta) = \frac{1 - e^{\frac{-\theta G(x;\beta)}{\alpha+\bar{\alpha}G(x;\beta)}}}{1 - e^{-\theta}} \qquad \beta,\alpha,\theta > 0, \qquad x > 0 \tag{3}$$

$$f(x;\alpha,\theta;\ \beta) = \frac{\theta\alpha g(x;\beta) e^{\frac{-\theta G(x;\beta)}{\alpha+\bar{\alpha}G(x;\beta)}}}{(1 - e^{-\theta})\,(\alpha + \bar{\alpha}G(x;\beta))^2} \tag{4}$$

And by Substituting the equation (1) in (3) we get the CDF of the new distribution (TEMO-R) and by deriving the CDF we get the PDF as follows.

$$F(x;\alpha,\theta,\beta) = \frac{1 - e^{\frac{-\theta\left(1-e^{-(\beta x)^2}\right)}{\alpha+\bar{\alpha}\left(1-e^{-(\beta x)^2}\right)}}}{1 - e^{-\theta}} \tag{5}$$

$$f(x;\alpha,\theta,\beta) = \frac{2\theta\,\alpha\beta^2\, e^{-(\beta x)^2}\, e^{\frac{-\theta\left(1-e^{-(\beta x)^2}\right)}{\alpha+\bar{\alpha}\left(1-e^{-(\beta x)^2}\right)}}}{(1 - e^{-\theta})[\alpha + \bar{\alpha}\left(1 - e^{-(\beta x)^2}\right)]^2} \tag{6}$$

The hazard function of the new distribution is:

$$h(x;\alpha,\theta,\beta)=\frac{\left\{\dfrac{2\theta\,\alpha\beta^{2}\,e^{-(\beta x)^{2}}\,e^{\frac{-\theta\left(1-e^{-(\beta x)^{2}}\right)}{\alpha+\overline{\alpha}\left(1-e^{-(\beta x)^{2}}\right)}}}{(1-e^{-\theta})[\alpha+\overline{\alpha}\left(1-e^{-(\beta x)^{2}}\right)]^{2}}\right\}}{1-\left\{\dfrac{1-e^{\frac{-\theta\left(1-e^{-(\beta x)^{2}}\right)}{\alpha+\overline{\alpha}\left(1-e^{-(\beta x)^{2}}\right)}}}{1-e^{-\theta}}\right\}} \qquad (7)$$

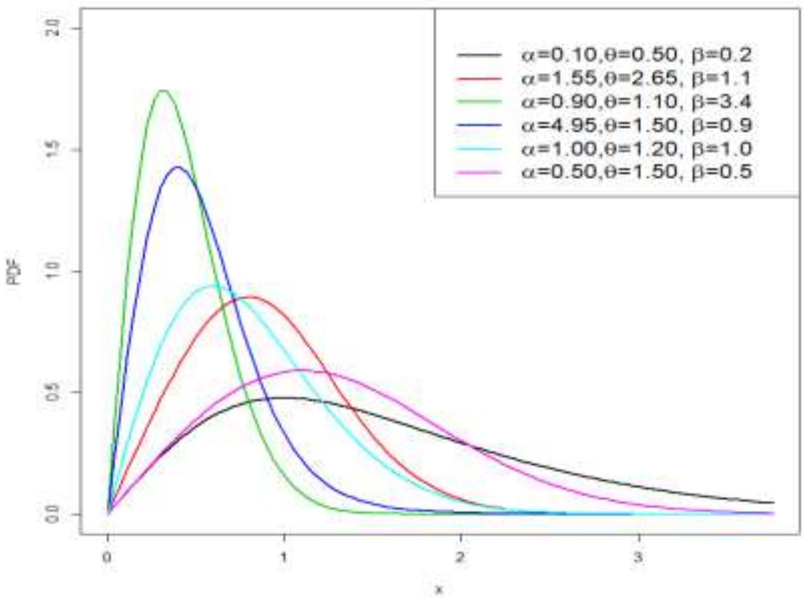

Figure (1): The pdf's of various TEMO-R distributions

Note that [0,1]Truncated Exponential Marshall Olkin Rayleigh (TEMO-R) is an extended model for analyze more complex data. That clearly is Rayleigh distribution is a special case for α=0.10. Figure 1 Show some of the possible shapes of the pdf of a new distribution for selected values of the parameters.

3. Expansion of PDF and CDF:

In this part, the PDF function will be reworked by decoding and expanding the cdf function and pdf function of the new distribution (TEMO-R), which is useful for studying some statistical properties. Let us consider the generalized binomial expansion and Exponential expansion as (Khaleel et al.2020) :

$$e^{-u}=\sum_{m=0}^{\infty}\frac{(-1)^{m}}{m!}\,u^{m},(1-u)^{a}=\sum_{k=o}^{\infty}(-1)^{k}\binom{a}{k}\,u^{k},(1-u)^{-a}=\sum_{k=o}^{\infty}\frac{\Gamma(a+k)}{\Gamma(a)k!}\,u^{k}$$

To simplify the PDF as simplify equation (4) in general as follows:

$$f(x;\alpha,\theta,\beta)=\frac{\theta\alpha g(x;\beta)e^{-\theta\left(\frac{G(x;\beta)}{\alpha+\bar{\alpha}G(x;\beta)}\right)}}{(1-e^{-\theta})\left(\alpha+\bar{\alpha}G(x;\beta)\right)^{2}}=\frac{\theta\alpha g(x;\beta)}{(1-e^{-\theta})}e^{-\theta\left(\frac{G(x;\beta)}{\alpha+\bar{\alpha}G(x;\beta)}\right)}\left(\alpha+\bar{\alpha}G(x;\beta)\right)^{-2}$$

By using Exponential expansion we get:

$$e^{-\theta\left(\frac{G(x;\beta)}{\alpha+\bar{\alpha}G(x;\beta)}\right)}=\sum_{m=0}^{\infty}\frac{(-1)^m\theta^m}{m!}\left(\frac{G(x;\beta)}{\alpha+\bar{\alpha}G(x;\beta)}\right)^m$$

$$=\sum_{m=0}^{\infty}\frac{(-1)^m\theta^m}{m!}\left(G(x;\beta)\right)^m\left(\alpha+\bar{\alpha}G(x;\beta)\right)^{-m}$$

After some algebra we get:

$$f(x;\alpha,\theta,\beta)=\sum_{m=0}^{\infty}\frac{\theta^{m+1}\alpha(-1)^m}{(1-e^{-\theta})m!}g(x;\beta)\left(G(x;\beta)\right)^m\left(\alpha+\bar{\alpha}G(x;\beta)\right)^{-(2+m)}$$

By using binomial series expansion we get:

$$\left(\alpha+\bar{\alpha}G(x;\beta)\right)^{-(2+m)}=\left(\alpha+(1-\alpha)\left(1-\bar{G}(x;\beta)\right)\right)^{-(2+m)}=\left(1-(1-\alpha)\bar{G}(x;\beta)\right)^{-(2+m)}$$

$$=\left(1-\bar{\alpha}\bar{G}(x;\beta)\right)^{-(2+m)}=\sum_{k=0}^{\infty}\frac{\Gamma(2+m+k)}{k!\,\Gamma(2+m)}\left(\bar{\alpha}\bar{G}(x;\beta)\right)^k$$

Then we have After some algebra:

$$f(x;\alpha,\theta,\beta)=\sum_{m=k=0}^{\infty}\frac{\theta^{m+1}\alpha(-1)^m(\bar{\alpha})^k}{(1-e^{-\theta})\Gamma(2+m)m!\,k!}g(x;\beta)\left(G(x;\beta)\right)^m(1-G(x;\beta))^k$$

Again, by using binomial series expansion we get:

$$(1-G(x;\beta))^k=\sum_{j=0}^{\infty}(-1)^j\binom{k}{j}\left(G(x;\beta)\right)^j$$

$$f(x;\alpha,\theta,\beta)=\sum_{m=k=j=0}^{\infty}\frac{\theta^{m+1}\alpha(-1)^{m+j}(\bar{\alpha})^k\binom{k}{j}}{(1-e^{-\theta})\Gamma(2+m)m!\,k!}g(x;\beta)\left(G(x;\beta)\right)^{m+j} \tag{8}$$

And by Substituting equation (1) and (2) in (8) we get:

$$f(x;\alpha,\theta,\beta)=\sum_{m=k=j=o}^{\infty}\frac{\theta^{m+1}\alpha(-1)^{m+j}(\bar{\alpha})^k\binom{k}{j}}{(1-e^{-\theta})\Gamma(2+m)m!\,k!}\{2\beta^2xe^{-(\beta x)^2}\}\{1-e^{-(\beta x)^2}\}^{m+j}$$

by using binomial series expansion we get:

$$\{1-e^{-(\beta x)^2}\}^{m+j}=\sum_{i=o}^{\infty}(-1)^i\binom{j+m}{i}\left(e^{-(\beta x)^2}\right)^i$$

Then we get:

$$f(x;\alpha,\theta,\beta)=\sum_{m=k=j=i=0}^{\infty}\frac{2\beta^2\theta^{m+1}\alpha(-1)^{m+j+i}(\bar{\alpha})^k\binom{k}{j}}{(1-e^{-\theta})\Gamma(2+m)m!k!}\binom{j+m}{i}x\left(e^{-(\beta x)^2}\right)^i\left(e^{-(\beta x)^2}\right)$$

$$f(x;\alpha,\theta,\beta)=\sum_{m=k=j=i=0}^{\infty}\psi_{m,k,j,i}\;x\,e^{[-(1+i)(\beta x)^2]} \tag{9}$$

where $\psi_{m,k,j,i}=\frac{2\beta^2\theta^{m+1}\alpha(-1)^{m+j+i}(\bar{\alpha})^k}{(1-e^{-\theta})\Gamma(2+m)m!k!}\binom{k}{j}\binom{j+m}{i}$

In the same way, we expand the CDF as follows:

$$F(x;\alpha,\theta,\beta)=\frac{1-\left\{\sum_{k=j=i=m=0}^{\infty}¥_{k,j,i,m}\quad e^{-i(\beta x)^2}\right\}}{1-e^{-\theta}} \tag{10}$$

Where $¥_{k,j,i,m}=\sum_{k=j=i=m=0}^{\infty}\frac{(-1)^{m+j+i}\Gamma(m+k)}{k!\,m!\,\Gamma(m)}\quad \theta^m(\overline{\alpha})^k\binom{k}{j}\binom{m+j}{i}$

4. Mathematical Properties:

In this section, we obtain some important mathematical properties of TEMO-R distribution.

4.1. Quantile Function:

The quantile function has a lot of uses in the statistics and because its shape is closed will help in determining the quarters of the distribution As well as generating values for TEMO-R distribution we use in simulation to find the flat (Moor's Kurtosis) and twisting (Bowley's Skewness) where these metrics are used to find distributions with twisting as well as distributions that do not have the third or fourth moment. the quantile function is found of the relationship:

$$Q(u)=F^{-1}(x)$$

From the Substituting equation 3 in the previous relationship and the conduct of some mathematical processes we get:

$$Q(u)=-\frac{1}{a}\left\{-\ln\left[1-\left\{\frac{-\alpha\, ln[1-u(1-e^{-\theta})]}{\theta+\overline{\alpha}\, ln[1-u(1-e^{-\theta})]}\right\}\right]\right\}^{\frac{1}{2}} \tag{11}$$

4.2. Moments:

It is one of the most important mathematical characteristics that have an important role in measuring and finding some statistical characteristics, such as standard deviation and coefficient of variation as well as average and variation. We can find r-degree moment to distribute TEMO-R from the following relationship:

$$M_r=E(x^r)=\int_{-\infty}^{\infty}x^r\, f(x;\alpha,\theta,\beta)\,dx$$

By Substituting equation (9) in the previous relationship we get:

$$\mu_r=\sum_{m=k=j=i=0}^{\infty}\Psi_{m,k,j,i}\int_0^{\infty}x^r\; x\; e^{-(1+i)(\beta x)^2}$$

Where $\psi_{m,k,j,i}=\frac{2\beta^2\theta^{m+1}\alpha(-1)^{m+j}(\overline{\alpha})^k}{(1-e^{-\theta})\Gamma(2+m)m!k!}(-1)^i\binom{j+m}{i}$

Now using the conversion $t=(\beta x)^2$

let $t=(\beta x)^2,\ x=\frac{\sqrt{t}}{\beta}\quad,\ dt=2\beta^2x\,dx\quad,dx=\frac{dt}{2\beta^2x}\quad,dx=\frac{dt}{2\beta\sqrt{t}}$

Then $\int_0^\infty x^{r+1}\, e^{-(1+i)(\beta x)^2} = \int_0^\infty (\frac{\sqrt{t}}{\beta})^{r+1} e^{-(1+i)t}\, \frac{dt}{2\beta t^{1/2}}$

$$= \frac{1}{2\,\beta^{r+2}} \int_0^\infty (t)^{r+1/2}\, t^{-1/2}\, e^{-(1+i)t} dt \;=\; \frac{\beta^{-(r+2)}}{2} \int_0^\infty (t)^{r/2}\; e^{-(1+i)t} dt$$

$$= \quad \frac{\beta^{-(r+2)}}{2}\;(1+i)^{\frac{r}{2}+1}\,\Gamma\left(1+\frac{r}{2}\right)$$

By Using this integration:

$$\int_0^\infty y^{\frac{-r}{2}} \exp[-2y]\, dy = (2)^{\frac{r}{2}+1}\,\Gamma\left(1-\frac{r}{2}\right) \quad , \qquad r < 2$$

$$\mu_r = \sum_{m=k=j=i=0}^{\infty} \psi_{m,k,j,i}\, \frac{\beta^{-(r+2)}}{2}\; (1+i)^{\frac{r}{2}+1}\,\Gamma\left(1+\frac{r}{2}\right) \tag{12}$$

4.3. Moment Generating function:

To find the moment-generating function of the new distribution TEMO-R for the continuous variable X, we use the following relationship:

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx}\, f(x,\alpha,\theta,\beta)dx \;=\; \int_0^\infty e^{tx}\, f(x,\alpha,\theta,\beta)dx$$

But $e^{tx} = \sum_{r=0}^{\infty} \frac{t^r x^r}{r!}$, then

$$= \int_0^\infty \sum_{r=0}^{\infty} \frac{t^r}{r!}\, x^r\, f(x,\alpha,\theta,\beta)dx \;= \sum_{r=0}^{\infty} \frac{t^r}{r!}\, E(x^r) \tag{13}$$

by Substituting equation (12) in (13) we get:

$$M_{x(t)} = \sum_{m=k=j=i=r=0}^{\infty} \psi_{m,k,j,i}\, \frac{\beta^{-(r+2)}}{2}\frac{t^r}{r!}\;\; (1+i)^{\frac{r}{2}+1}\,\Gamma\left(1+\frac{r}{2}\right) \tag{14}$$

4.4. Order statistics:

Statistics are used in the reliability test to predict the failure of future elements, as well as in quality control and are important for use in many applications in the field of life such as finding higher and lower value. let $X_1, X_2, \ldots X_n$ be n-size random sample of the TEMO-R distribution that has a cumulative function in equation (10) as well as a probability density function in equation (9). Their ranked value is referred to as $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$, and we can obtain the ranked statistics from the following relationship.

$$f_{s:n}(x) = \frac{n!}{(s-1)!\,(n-s)!}\, f(x)[F(x)]^{s-1}[1-F(x)]^{n-s}$$

$$f_{s:n}(x) = \sum_{q=0}^{\infty} \frac{n!\,(-1)^q}{(s-1)!\,(n-s)!}\; \binom{n-s}{q}\, f(x)[F(x)]^{s+q-1} \tag{15}$$

When Substituting equations (9) and (10) in (15) we get:

$$f_{s:n}(x) = \sum_{q=0}^{\infty} \frac{n!\,(-1)^q}{(s-1)!\,(n-s)!} \binom{n-s}{q} \sum_{m=k=j=i=0}^{\infty} \psi_{m,k,j,i}\ x e^{-(1+i)(\beta x)^2}$$

$$* \left[\frac{1-\left\{\sum_{k=j=i=m=0}^{\infty} ¥_{k,j,i,m}\quad e^{-i(\beta x)^2}\right\}}{1-e^{-\theta}}\right]^{s+q-1} \tag{16}$$

We obtain the density of the smallest order statistic as:

$$f_{1:n}(x) = \sum_{q=0}^{\infty} n \binom{n-1}{q} \left[\sum_{m=k=j=i=0}^{\infty} \psi_{m,k,j,i}\ x e^{-(1+i)(\beta x)^2}\right]$$

$$* \left[\frac{1-\left\{\sum_{k=j=i=m=0}^{\infty} ¥_{k,j,i,m}\quad e^{-i(\beta x)^2}\right\}}{1-e^{-\theta}}\right]^{q}$$

we obtain the density of the largest order statistic as:

$$f_{n:n}(x) = \sum_{q=0}^{\infty} n \left[\sum_{m=k=j=i=0}^{\infty} \psi_{m,k,j,i}\ x e^{-(1+i)(\beta x)^2}\right]$$

$$* \left[\frac{1-\left\{\sum_{k=j=i=m=0}^{\infty} ¥_{k,j,i,m}\quad e^{-i(\beta x)^2}\right\}}{1-e^{-\theta}}\right]^{n+q-1}$$

4.5. Entropy:

The Entropy refers to the amount of uncertainty of a random variable and oscillation for any phenomenon studied. and it applies in the field of statistics (statistical inference, problems of determining distribution) as well as in the field of science and economy. Then the Renyi Entropy can be obtained by the relationship as.

$$I_R(\rho) = \frac{1}{1-\rho} \log \int_0^{\infty} f(x,\alpha,\theta,\beta)^{\rho}\ dx \qquad ,where \quad \rho > 0, \rho \neq 0$$

And from the Substituting equation (9) in the previous relationship we get:

$$I_R(\rho) = \frac{1}{1-\rho} \log \int_0^{\infty} \sum_{m=k=j=i=0}^{\infty} \psi_{m,k,j,i} \quad [x e^{-(1+i)(\beta x)^2}]^{\rho}\ dx$$

$$I_R(\rho) = \frac{1}{1-\rho} \log \sum_{m=k=j=i=0}^{\infty} \psi_{m,k,j,i} \int_0^{\infty} x^{\rho} \quad e^{-\rho(1+i)(\beta x)^2}\ \mathrm{dx}$$

Now using the conversion $t = (\rho + i\rho)(\beta x)^2$

Let $t = (\rho + i\rho)(\beta x)^2$, $x = \frac{\sqrt{t}}{\beta(\rho+i\rho)^{1/2}}$ $,dt = 2\beta^2 x(\rho+i\rho)dx\,,\ dx = \frac{(\rho+i\rho)^{1/2}\,dt}{2\beta\sqrt{t}(\rho+i\rho)}$

$$I_R(\rho) = \frac{1}{1-\rho} \log \sum_{m=k=j=i=0}^{\infty} \psi_{m,k,j,i} \int_0^{\infty} [\frac{t^{1/2}}{\beta(\rho+i\rho)^{1/2}}]^{\rho} e^{-t}\ e^{-t}\ \frac{(\rho+i\rho)^{1/2}}{2\beta t^{1/2}\,(\rho+i\rho)}\ \mathrm{dt}$$

$$I_R(\rho)=\frac{1}{1-\rho}\log\sum_{m=k=j=i=0}^{\infty}\psi_{m,k,j,i}\frac{1}{2\beta^{\rho+1}(\rho+i\rho)^{\frac{\rho-1}{2}}}\int_0^{\infty}t^{\frac{\rho-1}{2}}e^{-t}dt$$

$$I_R(\rho)=\frac{1}{1-\rho}\log\sum_{m=k=j=i=0}^{\infty}\psi_{m,k,j,i}\frac{1}{2\beta^{\rho+1}(\rho+i\rho)^{\frac{\rho-1}{2}}}\Gamma\left(\frac{\rho+1}{2}\right) \quad (17)$$

5. Parameter estimation:

In this section we will estimate the parameters of the new distribution partial derivatives of the function of the maximum likelihood method of distribution, so that we have $X_1, X_2, \dots X_n$ a random sample of the size n of the new distribution TEMO-R with the unknown parameters vector (α,ɵ, β)$^{\tau}$.

$$f(x,\alpha,\theta,\beta)=\frac{2\theta\alpha\beta^2 e^{-(\beta x)^2}\exp\left[\frac{-\theta\left(1-e^{-(\beta x)^2}\right)}{\alpha+\bar{\alpha}\left(1-e^{-(\beta x)^2}\right)}\right]}{(1-e^{-\theta})[\alpha+\bar{\alpha}\left(1-e^{-(\beta x)^2}\right)]^2}$$

$$L=L(\alpha,ɵ,\beta/X_i)=\prod_{i=1}^{n}f(x,\alpha,\theta,\beta)$$

The log-likelihood function is given as.

$$Ln(L)=nln(2)+nln(\theta)+nln(\alpha)+nln(\beta^2)-\sum_{i=1}^{n}\beta^2X_i^{\,2}-\sum_{i=1}^{n}\frac{\theta\left(1-e^{-(\beta x_i)^2}\right)}{\alpha+(1-\alpha)[1-e^{-(\beta x_i)^2}]}$$
$$-nln\left(1-e^{-\theta}\right)-2\ln\left[\alpha+(1-\alpha)\left(1-e^{-(\beta x)^2}\right)\right]$$

Now we find the partial derivatives of TEMOR distribution features, which are β, α, ɵ.

$$\frac{\partial\ln(L)}{\partial\theta}=\frac{n}{\theta}-\sum_{i=1}^{n}\frac{\left(1-e^{-(\beta x_i)^2}\right)}{\alpha+(1-\alpha)\left(1-e^{-(\alpha\beta)^2}\right)}-\frac{ne^{-\theta}}{(1-e^{-\theta})}$$

$$\frac{\partial\ln(L)}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\frac{\theta\left(1-e^{-(\beta x_i)^2}\right)\left(e^{-(\beta x_i)^2}\right)}{\left[\alpha+(1-\alpha)[1-e^{-(\beta x_i)^2}]\right]^2}-\sum_{i=1}^{n}\frac{2e^{-(\beta x_i)^2}}{\alpha+(1-\alpha)\left(1-e^{-(\beta x_i)^2}\right)}$$

$$\frac{\partial\ln(L)}{\partial\beta}=\frac{2\beta}{n}-\sum_{i=1}^{n}2\beta\,x_i^{\,2}$$
$$-\sum_{i=1}^{n}\frac{\theta\left(1-e^{-(\beta x_i)^2}\right)\left[(1-\alpha)2\beta x_i^{\,2}e^{-(\beta x_i)^2}\right]-\left|\alpha+(1-\alpha)[1-e^{-(\beta x_i)^2}]\right|2\beta\theta x_i^{\,2}e^{-(\beta x_i)^2}}{\left\{\left[\alpha+(1-\alpha)[1-e^{-(\beta x_i)^2}]\right]\right\}^2}$$
$$-\sum_{i=1}^{n}\frac{4(1-\alpha)\beta x_i^{\,2}e^{-(\beta x_i)^2}}{\left[\alpha+(1-\alpha)\left(1-e^{-(\beta x_i)^2}\right)\right]}$$

$$\frac{\partial\ln(L)}{\partial\theta}=\frac{\partial\ln(L)}{\partial\alpha}=\frac{\partial\ln(L)}{\partial\beta}=0$$

Now we not when the partial derives are equal to zero, it is difficult to solve them manually, so there are several statistical programs through which the function can be maximized and an approximate solution, from these programs OX, SAS, (R), as well as mathematical.

6. Application:

In this section we review a real data set for the purpose of demonstrating the potential of the new model, and comparing it with several distributions such as Beta Rayleigh distribution (BR) and the PDF of this distribution is

$$f_{BR}(x;\beta,a,b)=\frac{2\beta^2 x}{\beta(a,b)}\left[1-e^{-(\beta x_i)^2}\right]^{a-1} e^{(b-2)(\beta x_i)^2} \qquad x,\beta,a,b>0$$

and Kumaraswamy Rayleigh distribution (KuR) and the PDF of this distribution is

$$f_{KuR}(x;\beta,a,b)=2ab\beta^2 x\left[1-e^{-(\beta x_i)^2}\right]^{a-1} e^{(b-2)(\beta x_i)^2} \qquad x,\beta,a,b>0$$

exponential generalized Rayleigh distribution (EGR) (New), and Weibull Rayleigh distribution (WR) (Merovci, F., & Elbatal, I. 2015), Gompertz Rayleigh distribution (GoR) and the PDF of this distribution is.

$$f_{GoR}(x;\beta,\theta,\alpha)=2\theta\beta^2 x\, e^{(-\alpha-2)(\beta x_i)^2}\, e^{\frac{\theta}{\alpha}\left[1-e^{-\alpha(\beta x_i)^2}\right]} \qquad x,\beta,\theta,\alpha>0$$

Rayleigh distribution (R) and it is PDf is the equation (1). The R program was used to calculate Negative Log Likelihood (NLL), Akaike Consistent Information Criteria (CAIC), Hanan and Quinn Information Criteria (HQIC), Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and quantification of parameter values according to the MLE method. Respectively as in the following equations:

$$BIC=-2l+k\log(n),\ AIC=-2l+2k,\ HQIC=2k\ln[\ln(n)]-2l,\ CAIC=AIC+\frac{2k(k+1)}{n-k-1}$$

Where the value (K) is Like the number of parameters of the distribution, value (n) Represent the sample size and (l) is the maximized value of the log-likelihood function under the considered model. and that Using R language program to find the best distribution that matches the data.

6.1. Data1:

This real data collection has been taken from (Smith, R. L., & Naylor, J. 1987). Consist of 63 observations of the strength of 1.5 cm of fiber glass, and has been obtained by workers at the National Physics Laboratory in the UK.

This data has already been analyzed by (Merovei et al., 2016), (Oguntunde et al., 2017), (Khaleel et al., 2018), (Oguntunde et al., 2018). The observations are:

(1.61, 1.63,1.61, 1.61, 1.62, 1.62, 1.67, 1.64, 1.66, 1.66, 1.66, 1.70, 1.68,1.68, 1.69, 1.70, 1.78, 1.73, 1.76, 1.76, 1.77, 1.89, 1.81, 1.82,1.84, 1.84, 2.00, 2.01, 2.24).

Table (1): Statistical Description of Data

Var	n	mean	sd	median	Min	Max	Skew	Kurtosis
	63	1.51	0.32	1.59	0.55	2.24	-0.88	0.8

As for kurtosis, its value is positive, meaning that the data with a thin flatness are close to a normal distribution, Note that the skew value is negative, which indicates that the data is skewed to the left. that is, the mean and median of the data are close to the value. The result is presented in Table (2).

Table (2): result of data1

Model	Est para	-LL	AIC	CAIC	BIC	HQIC
TEMOR	1.0916 62.926 1.9453	13.02	32.05	32.46	38.48	34.58
BR	5.0992 17.279 0.1119	20.88	47.77	48.17	54.20	50.30
KuR	3.3801 42.958 0.1515	16.26	38.58	38.93	44.95	41.05
EGR	0.6311 5.4860 1.5287	23.92	53.85	54.26	60.28	56.38
WR	2.8903 3.8582 1.4555	15.20	36.41	36.82	42.84	38.94
GoR	0.0875 1.1635 0.8282	16.49	38.99	39.40	45.42	41.52
R	0.4212	49.79	101.58	101.64	103.72	102.42

Table (2) notes that the new distribution has very good places compared to other distributions because it has the lowest standards values (-LL, AIC, CAIC, BIC, HQIC). The new distribution is the most appropriate for the real data on display. To verify the results obtained, the chart of the data from Table (2) with other distributions compared to it is displayed in Figure (2), and the experimental graph of the new distribution is shown in Figure 3.

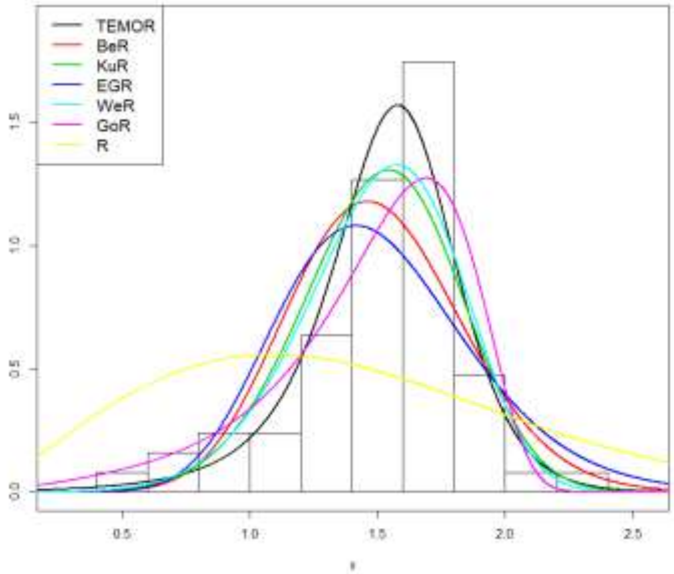

Figure (2): Histogram plot of the dataset1 with the compared distributions

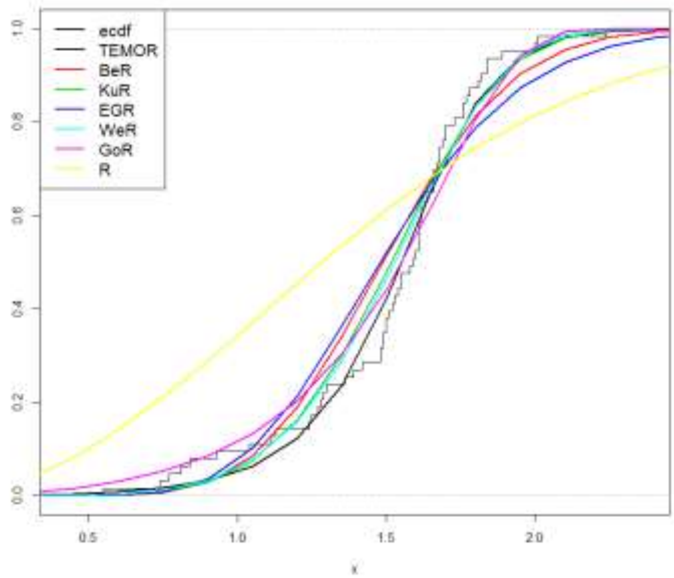

Figure (3): Empirical cdf of the dataset1 with the compared distributions

6.2. Data2:

The set of data2 used consists of the fatigue fracture life of the 373 Kevlar / epoxy that is subjected to continuous pressure at a stress level of 90% until all have failed. Data set 2 was previously extracted from (Owoloko et al., 2015). The observations are:

(0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960).

Table (3): Statistical Description of Data

Var	n	mean	sd	median	Min	Max	Skew	Kurtosis
	76	1.96	1.57	1.74	0.0251	9.096	1.94	4.95

As for kurtosis, it is of positive value, which indicates that the data with thin flatness is close to normal distribution. Note that the skewness has a positive value, indicating that the data is skewed to the right.

Notes The result is presented in Table (4).

Table (4): Results of data2

Model	Est para	-LL	AIC	CAIC	BIC	HQIC
TEMOR	2.430 0.235 0.031	122.3	250.7	251.1	257.7	253.5
BR	0.540 4.809 0.018	124.8	255.7	256.0	262.7	258.5
KuR	0.619 5.856 0.010	123.3	252.6	253.0	295.4	255.3
EGR	0.864 0.532 0.115	125.5	257.0	275.3	263.9	259.7
WR	0.662 0.958 0.210	122.5	251.9	251.4	258.2	253.8
GoR	0.997 0.198 0.255	125.8	257.7	258.0	264.8	260.5
R	1.159	137.3	276.6	276.6	278.9	277.5

The newly developed TEMO-R shows very good potentials in Table (4) since it has the lowest values for standard NLL, AIC, CAIC, BIC and HQIC. To further validate the results obtained, the histogram of data set 2 with the distributions compared in Figure (4) and the corresponding experimental cdf plot in Figure 5 are presented.

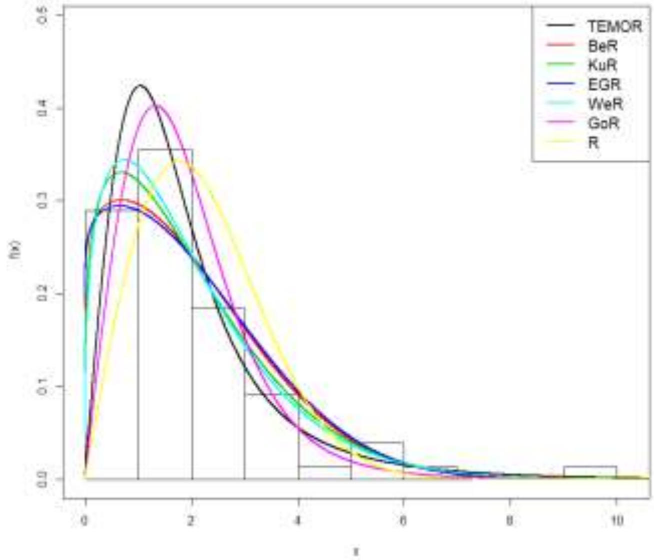

Figure (4): Histogram plot of the dataset 2 with the compared distributions

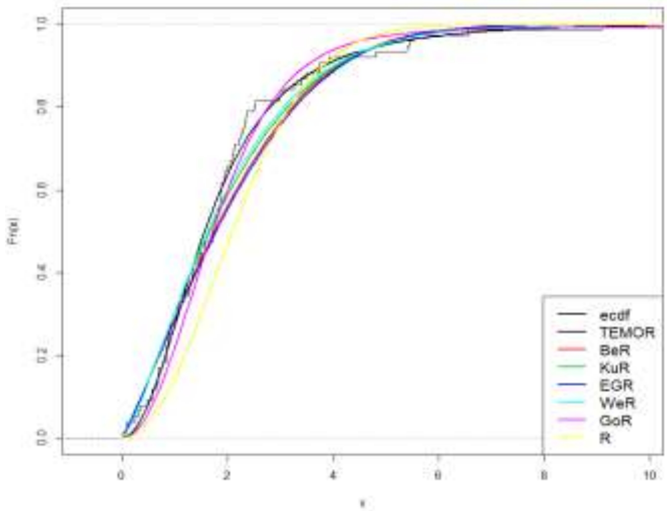

Figure (5): Empirical cdf of the dataset2 with the compared distributions

7. Conclusion:

In this research, a new called the [0,1] Truncated Exponential Marshall Olkin Rayleigh distribution (TEMO-R) three-parameter distribution was introduced. Some of the mathematical characteristics of the model TEMO-R were discussed such as Quantile Function, the moments, the moments -generating function, entropy, order statistics. and find the capabilities of the distribution parameters by maximum likelihood method. This model TEMOR contains some special distributions. The importance of this model was illustrated by its application to real data, as it was found that this model is more flexible compared to other distributions as a result of obtaining the lowest value for(-LL, AIC, CAIC, BIC, HQIC). Therefore, this model is more flexible in modeling real data than many distributions.

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